The Hidden Triangle Problem

Primary standard addressed:

A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate place, often forming a curve (which could be a line).

Additional standards addressed:

A-REI (first cluster) Understand solving equations as a process of reasoning and explain the reasoning.

A-SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$*.*

HIDDEN TRIANGLE

This problem is about the graph of the following equation in the *xy* coordinate plane:

$$(x - 1)^2(y - 1)^2 = (x^2 - 1)(y^2 - 1)$$

a) Show that the graph of the equation includes the point (1, 17):
b) Show that the graph also includes the point (23, 1).
c) Show that any point on the line *x* = 1 belongs to the graph.
d) Show that any point on the line *y* = 1 belongs to the graph.
e) Show that any point on the line *y* = −*x* belongs to the graph.
f) (Optional) Show that if a point is <u>not</u> on any of the lines *x* = 1, *y* = 1, or *y* = −*x*, then the point does <u>not</u> belong to the graph. (Suggestion: Begin by factoring the right-hand side of the equation completely.)
g) Use technology to graph the equation. For example, go online to <u>www.wolframalpha.com</u> and type the following command:

ContourPlot[(x-1)^2*(y-1)^2==(x^2-1)*(y^2-1), {x, -2, 2}, {y, -2, 2}].