Queensland University of Technology

Centre for Mathematics, Science and Technology Education

RICH TASKS PROJECT

Integrating mathematics outcomes with Rich Tasks within a Productive Pedagogies framework

REPORT

An expert analysis of the Rich Tasks in relation to teaching mathematics Years 1-9

Prepared for the Assessment and New Basics Branch,

Education Queensland

April 2003

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Executive Summary

Background to the report

New Basics is one of Education Queensland's most significant educational initiative to date. It is a futures-oriented strategic philosophy that aims to improve educational outcomes through a complete re-examination and re-alignment of curricular, pedagogical and assessment philosophy and practices.
One of the three pillars of New Basics (the other two being the New Basics and Productive Pedagogies) is Rich Tasks. The Rich Tasks are sets of substantive, life-like activities that students undertake over a three-year period, culminating in a demonstration or performance or product in the final year of the three-year period. Rich Tasks are thus designed to connect students to the real world in substantive and meaningful ways which, hopefully, will engage and also challenge students intellectually (Assessment & New Basics Branch b, 2000). As teaching, learning and assessment devices, the Rich Tasks present teachers and students with completely new ways of teaching and learning. One important question is whether the problem-based nature of Rich Tasks and the transdisciplinary approach required for their successful completion compromise the acquisition of knowledge and skills in the individual disciplines / fields of knowledge. This report is concerned with answering this question with specific reference to mathematics. It is written from the perspective of mathematics education by a team of three mathematics educators and researchers from the Centre of Mathematics, Science and Technology Education, Queensland University of Technology, Brisbane. During the course of the investigation, the team visited and interviewed teachers in two schools.

Summary of findings

The investigation found that the Rich Tasks do not cover all the aspects of mathematics that the authors consider fundamental to mathematics learning. They (the Tasks) place heavy emphasis on certain aspects of mathematics and these aspects are present in most Tasks. At the same time, there are important omissions which, if not remedied, will be detrimental to mathematics learning.

Specifically, the investigation found that:

(1) The Rich Tasks are inadequate on their own to teach, assess or report on mathematics learning. Analysis shows that major process and content ideas are not covered by the Tasks.

(2) Although considering mathematics as prerequisite for, embedded in and associated with Rich Tasks increases the mathematics that can be taught in relation to the Tasks, there are major components of mathematics that must be taught outside of the Rich Tasks.

(3) This out-of-Task mathematics needs to focus predominantly on concept development that leads to abstraction or mathematics topics that are not normally central to investigation and presentation.

(4) The Rich Tasks can be integrated with mathematics if out-of-Task, prerequisite, embedded and associated mathematics is sequenced to enable both Tasks and mathematics to be developed.

(5) This integration also requires that Rich Tasks be built into the curriculum and part of a general approach to teaching and learning that focuses on authentic situations and intellectual engagement.

Section 1:
The focus of the report

This report is an expert analysis of the Education Queensland Rich Tasks as an aid to teaching mathematics Years 1-9. It was undertaken for the Assessment and New Basics Branch of Education Queensland as part of the Branch's mission to generate research data on the issue of mathematics teaching with the Rich Tasks.

1.1 Background to the analysis

The Rich Tasks were developed as part of a new curriculum approach called New Basics which Education Queensland felt would provide students with substantive, meaningful and authentic learning practices that better prepare students for the changing nature of work in an information-oriented society. The Rich Tasks were designed to be implemented in three parts, namely, Years 1 to 3, Years 4 to 6 and Years 7 to 9.

Early informal feedback and anecdotal evidence from the implementation of the Rich Tasks in Queensland state schools found that there might be difficulties with teaching mathematics through the Rich Tasks that had the potential to interfere with the take up of the Tasks. Some teachers were finding it difficult to use the Tasks to teach and assess mathematics learning. There were indications that schools were not integrating the Rich Tasks with their mathematics teaching to the same extent they were integrating it with teaching in other disciplines and were seeing the Rich Tasks as an impediment to what the teachers in the schools believed was effective mathematics teaching.

Therefore, the authors were asked to analyse the Tasks in terms of our knowledge of mathematics curriculum, mathematics teaching and learning, and mathematics assessment and evaluation for Years 1 to 9 of school, and to suggest ways in which the Tasks could be best integrated with mathematics teaching and learning to maximise integration of mathematics into the Tasks and students' learning outcomes. In particular, we were asked to find ways in which the Tasks could be used to teach and assess a wider range of mathematics content and in which a wider range of mathematics content could be associated with the Tasks.

To meet this request, this report comprises six sections as described in Section 1.2.

1.2 The brief for the analysis

The initial brief provided by the Assessment and New Basics Branch of Education Queensland was to complete the five tasks below.

(1) comparison of the Rich Tasks with the Queensland 1-10 Mathematics syllabus and identification of the mathematical concepts that could be used within the Rich Tasks and which the Rich Tasks can assess and measure;

(2) theoretical analysis of the mathematical knowledge (content, processes, strategies and meta-strategies) underpinning the Rich Tasks and the identification of the mathematical requirements which students need to successfully address the Tasks;

(3) determination of innovative and creative ways in which mathematical outcomes are explicitly integrated into the Rich Tasks and in which the Rich Tasks can meet and reinforce other identified outcomes;

(4) publication of a report outlining findings from the above tasks; and

(5) development of a teacher resource package, in a form suitable to be sent to teachers, which provides advice as to how to integrate mathematics into the Rich Tasks.

The comparison of the Rich Tasks with the Queensland 1-10 Mathematics syllabus and identification of the mathematical concepts that could be used within the Rich Tasks and which the Rich Tasks can assess and measure was completed and presented to the Assessment and New Basics Branch in December 2002. As a consequence of this presentation, the brief for the final report was modified to focus more directly on the question how Rich Tasks can be an effective vehicle for teaching, learning and assessing mathematics, and to

answer this question independently of the particular outcomes in the new Queensland 1-10 Mathematics syllabus. To meet this modified brief, the authors were requested to do the following four tasks.

(1) development of a mathematics "summary syllabus" representing best international practice in terms of mathematics teaching and learning in Years 1 to 9;

(2) theoretical analysis of knowledge from this "summary syllabus" required for and motivated by the Rich Tasks;

(3) development of a plan for integrating the Rich Tasks with the "summary syllabus" for the benefit of mathematics teaching, learning and assessment; and

(4) publication of a report outlining findings from the above tasks.

The analysis of the Rich Tasks presented in this report is to meet these four modified tasks.

The last part of the initial brief was the development of a resource package for teachers. This is not contained in this report and will depend on the acceptance of the report by the Assessment and New Basics Branch. This will ensure that any package is based on a design and concept that meets the requirements of the Branch.

1.3 Structure of the report

This report is written as an expert analysis, based on the authors' knowledge and experience as mathematics educators and researchers. While we have identified the major works that have influenced our positions with regard to mathematics and to mathematics teaching, learning and assessment, there has been no attempt to provide an extensive literature review. Thus, although the report is written from the perspective of research, the research findings are integrated within the knowledge of the authors as experts on mathematics teaching, learning and assessment. This "research" is then presented in an integrated form within the mathematics "summary syllabus" generated for the analysis, and the creative ways in which mathematics is integrated within the Rich Tasks.

The report contains six sections.

(1) Section 1 discusses the focus of the report, contains the Brief and outlines the structure of the report.

(2) Section 2 provides the context and background to the report. It describes each of the Tasks and how they relate to the New Basics referents.

(3) Section 3 provides the framework for mathematics and its teaching, learning and assessment that is the basis for the analysis, and describes the major mathematics ideas that should be covered in Years 1 to 3, Years 4 to 6 and Years 7 to 9. It provides a "summary syllabus" for teaching mathematics within the three levels.

(4) Section 4 contains a discussion of the major issues associated with mathematics teaching through the Rich Tasks, and reports on two case studies showing how these issues are tackled in two very different ways by two schools. It describes the categories of mathematics that are used in later sections to describe the mathematics in the Rich Tasks.

(5) Section 5 identifies the mathematics ideas contained in each Rich Task and audits the Tasks against the mathematics knowledge contained in the "summary syllabus". It lists the mathematics ideas that underlie (are prerequisites for) the Tasks, are embedded in the Tasks, and may be associated with the Tasks.

(6) Section 6 suggests ways in which mathematics teaching and the Rich Tasks can be programmed across the three-year intervals that have the potential to enable Rich Tasks to be integrated into the teaching, learning and assessment of mathematics.

Section 2:
The Rich Tasks

2.1 The context of Rich Tasks

The Rich Tasks are part of New Basics, an educational innovation introduced by the Queensland government as a response to questions raised (see, e.g., ANTA, 1995; ANTA, 1998; Finn, 1991; Mayer, 1992; MCEETYA, 1997; NCVER, 2000; Wiltshire, 1994) about the capacity of the education system to prepare young people for the 21st century, when many social, economic and technological factors have profoundly altered the meaning and nature of work, and the role of work in a person's life. These factors are summarised below.

Increasing school retention rates

The increase in school retention rates in Australia as a whole, and Queensland specifically, is well documented. Australia's school retention rates have risen dramatically over the last 30 years – 33% in the 1970s to 73% in 2002 (Queensland Government, 2002). The Queensland government has set 88% as the target retention rate to be achieved by 2010 (Queensland Government, 2002). At the same time that students are staying at school longer, their post-school destinations have also significantly diversified. It is no longer the case that Year 12 is a preparatory year for university. Only just over 30% of Queensland's Year 12 leavers go on to universities (QTAC, 2001) (it could be argued that more students would like to go to study at university if there were more university places available but that's another story). The rest of the students attend technical and vocational courses offered by TAFE institutes or private providers, enter the work force or look for work. As the size and diversity of the student population increase, managing student behaviour becomes a major concern. Some students appear dissatisfied with school routine, disengaged from learning and unchallenged by classroom tasks they regard as irrelevant (Education Queensland, 2000). Numerous reports (see, e.g., Mayer, 1992; Pitman, 2002; Wiltshire, 1994) have attempted to address these issues, suggesting fundamental changes to both the school curriculum and school organisation in order to engage students and provide them with appropriate and meaningful learning experiences.

The changing meaning and nature of work

For those young people who are not pursuing further training or education, the prospects of full-time employment are not highly positive (CURASS, 1992; Kinsman, 1993; Shelldrake, 1998). The youth employment market has virtually collapsed, with the unemployment rate among Year 12 school leavers in Australia sitting at around 30% (Queensland Government, 2002). Various government enquiries and reports (e.g. Finn, 1991; Mayer, 1992; MCEETYA, 1997; Wiltshire, 1994) have attempted to look at the underlying causes of youth unemployment, and suggested ways to make young people more employable. While there has been a tendency to blame the problem on the educational system, it is nonetheless true that the nature and meaning of work have changed significantly in the last two or three decades, and that such changes call for a re-examination of the ways young people are prepared for life after school.

There are at least three aspects of the world of work where profound changes have been documented:

- the kinds of work to be undertaken;
- requirements for work performance; and
- engagement with the workplace or work situation.

<u>Changes to the kinds of work to be undertaken</u>

Many writers on the topic agree that work and the meaning attached to it have changed significantly since the beginning of post-industrialism in the early 1980s. For example, Tinkler et al (1996) argued that there has been a shift in the modes of production from the Fordist method of mass production, to post-Fordist production, with multi-skilled workers producing a more diverse range of products. The application of technology and work redesign to manufacturing industry has not only brought about changes in that sector's production methods, it has also reduced the work force for the sector. Consequently, much routine and

unskilled work has disappeared while, at the same time, the remaining work force is expected to perform a wider range of tasks and produce a more diverse range of products (Tinkler et al, 1996).

At the same time that the nature of some jobs has changed significantly, the mix or blend of work has also changed. Data from the Australian Bureau of Statistics (cited in Faculty of Education, 2002) indicate a decline in certain occupational categories such as Manufacturing, Agriculture and Transport, with increases in other categories (Finance, Property and Business Services, Construction and Retail). Reich (1991) referred to a powerful elite class of symbolic analysts who would dominate technology-driven economies, particularly in such areas as media, financial services and corporate sectors. It is hardly surprising that in their analysis of the "new work order", Gee, Hull and Lankshear (1998, cited in Education Queensland 2000) described the emergence of an elite of critical thinkers engaged in problem resolution and strategic planning, a middle order of information-based and service workers, and an underclass struggling to deal with technological and economic change (Education Queensland, 2000). The increasing predominance of all forms of information technology and communication has to be acknowledged and taken into consideration as education systems prepare our young people for the future.

Changes in requirements for work performance

As the nature of some jobs has changed significantly so have expectations of performance at work. Such expectations come from a plethora of factors. Advances in communications technologies have made the global village a reality rather than an abstract concept (OECD, 1996a; OECD, 1996b; Tinkler et al, 1996). In the process, governments of all political persuasions have embraced globalisation and deregulation with a degree of vigour (Carnoy, 1999; Forrester, Payne and Ward, 1995) and Australia is no exception. Where Australian industry and workers used to be protected by both geographical distances and trade barriers in the form of import quotas and high tariff, deregulation and globalisation have intensified competitiveness and placed more pressure on workers to increase their productivity. At the same time, a deregulated "flexible" and globalised labour market means that business and industry are able to look overseas to reduce labour costs in an effort to increase their comparative advantage.

While the new modes of production require workers to acquire new knowledge and develop higher-order skills and processes, this expectation has not always been matched by opportunities to learn the new skills. Billett (2000) describes this dilemma for workers well when he referred to the situation where the increased use of technology in work applications rendered invisible the knowledge required for and to understand its operation. Consequently, this knowledge is more difficult to learn and deploy because its accessibility is limited, particularly for casual and contract workers (Billett, 2000; Carnoy, 1999; Pütz, 2002). At the same time, there is increasing support – at least by government at both state and federal levels – for the view that it is an individual's responsibility to engage in lifelong learning to maintain currency of his/her employment-related skills (Bagnall, 2000). This requires that individual be autonomous learners, possessing skills that are transferable to novel contexts and situations.

Changing ways of engaging with work

One particular feature of modern-day Australia is the fast disappearance of the notion that employment is guaranteed to all, and that a person will have a job for life (New Basics Branch, 2000 b; Gibb, 1999). Sandra Yates, Chair of Saatchi and Saatchi, Australia, and Chair of the NSW TAFE Commission argued:

> It seems to me that we are still clinging to the notion that work must be central to people's lives–and yet business has already demonstrated in the most brutal of ways that it has no use for older or younger workers. In Australia today there is no more important task than job creation. (Gibb, 1999, p. 2)

Yates (Gibb, 1999) went on to argue that in the last 10 years there have been around 3.3 million retrenchments, yet Australia is creating jobs at the rate of only around 100,000 per year–nowhere near replacement for the jobs that have been lost. One of the effects of the higher rate of unemployment is significant *underemployment* especially among youth and middle-aged men (Gibb, 1999).

Implications for the educational system

It is quite clear that work is no longer guaranteed for all and is no longer central to the lives of many Australians. Even among those who are or will be employed, the world of work contains a degree of instability and uncertainty not experienced by the previous generations of workers. It has been estimated (Gibb, 1999) that today's average young Australian can expect to have six significant jobs in his/her working

life and be sacked from at least one. As young Australians are faced with these new demands and uncertainties for the future, governments of both political persuasions have been struggling to reshape the schooling system to provide an education that can sustain people in work as well as non-work situations. Central to many of these initiatives have been attempts to identify the kinds of skills that future workers will be requiring, and, therefore, by default, the kinds of skills that today's school children will have to develop (e.g., Finn, 1991; Mayer, 1992; Wiltshire, 1994). Occasionally, an attempt is made (Mayer, 1992) to link the employment-related skills to the non-work aspects of an individual's life. The central tenet remains, however, firmly rooted in an economic rationalist base (Dawkins, 1988).

The Federal government's strategies to reshape education in the country have been mainly through policies to converge vocational and general education, and to introduce a national curriculum. These initiatives have had a difficult passage with the states for two main reasons. First, there is the states' suspicion, if not hostility, of what they see as the Federal government's encroachment into state constitutional area of responsibility. Philosophically, opponents to the national curriculum agenda have argued that there can be no consensus on what constitutes ***the*** set of desirable skills for school children (Wiltshire, 1994). Determination of such a set must, by necessity, be based on a set of values (ACE, 2002; Wiltshire, 1994) which, in Australia's pluralistic and multicultural society, is objectionable to certain sections of the society.

Birth of New Basics in Queensland

It was against this background that New Basics was unveiled in Queensland in 2000. The initiative was presented to the Queensland people as

> …a bold and exciting undertaking by Education Queensland to prepare our students for the future. It deals with new student identities, new economies and workplaces, new technologies, diverse communities and complex cultures. The New Basics is about improving the learning outcomes of our students. It allows community members, teachers and students to work together to ensure that the richness and relevance of students' academic and social growth is enhanced. (http://www.education.qld.gov.au/corporate/newbasics/).

The New Basics Technical Paper (Education Queensland, 2000) which lays the theoretical and philosophical foundations for New Basics, describes the movement thus:

> Clusters or families of practices required for *contemporary* and *future* social, cultutral and economic life worlds. They describe the interactive requirements of identity, communications and social relations, work and citizenship, consumption and leisure. (Education Queensland, 2000, p. 114, emphasis added)

The architects of New Basics argue that the education system's response to the new world order must embrace all aspects of schooling: curriculum, pedagogy and assessment. Previous attempts at educational reforms had tended to focus on one particular aspect of schooling: curriculum – as is the case with the national curriculum agenda movement and the reconceptualisation of syllabuses carried out by the the Queensland School Curriculum Council; assessment (early intervention via the Year 2 Diagnostic Net, National Benchmark Testing) or pedagogy (Education Queensland's Excellence in Teaching program). They argue that what is required is a movement that reconceptualises all aspects of schooling (Education Queensland, 2000).

What New Basics promises to deliver is a quality education that will prepare students well for their post-school options and life-long learning. The three pillars on which New Basics rests are:

- the new basics (what is taught) – curriculum reform;
- productive pedagogies (how it is taught) – re-examination of teaching and learning practices; and
- Rich Tasks (how students show it) – a new way of assessing students.

What is taught at school should relate to what today's students need to know to acquit themselves well for life after school. The "old basics" of the three Rs are no longer adequate. New Basics identifies four clusters or families of practice and knowledge:

- *Life pathways and social futures*: Who am I and where am I going?
- *Multiliteracies and communication media*: How do *I make sense of and communicate with the world?

- *Active citizenship*: What are my rights and responsibilities in communities, cultures and economies?
- *Environments and technologies*: How do I describe, analyse and shape the world around me?

(Education Queensland, 2000)

How the New Basics are taught relates to the kind of classroom activities and interactions that engage students, deliver improved student outcomes and encourage higher-order and independent thinking. It focuses on both authentic and productive pedagogies. A *transdisciplinary* approach – which Education Queensland argues is not the same as *multi-disciplininary* (New Basics Branch a, 2000) – is followed that allows students to demonstrate their intellectual attainment unshackled by the bounds of the traditional disciplines. In this way schooling can be viewed as a total and purposeful experience.

The issue of how students show what they have learnt forms the third pillar of New Basics. Discussion papers from Education Queensland (Education Queensland, 2000; New Basics Branch, 2000 a & 2000 b) refer to the need to have assessment that are authentic in the sense that they mean something to students and the community in which they live. There is also the need to have assessment which is both credible and valid and which affords students the opportunity to demonstrate their rich learning experiences. Rich Tasks, which require students to demonstrate not only achievement across a number of disciplines but also the ability to employ their knowledge and skills in life-like problems and situations, are the New Basics's answer to this question.

The implementation plan for New Basics called for a trial over a four-year period with 38 schools throughout Queensland, accomnpanied by a large-scale formative and summative, qualitative and quantitative study of the trial (Education Queensland, 2000). The trial and research projects are monitored by a reference group comprising representatives of all the stakeholders.

Thus, what the *theory* of New Basics attempts to do is to directly address the issue of relevance of the education system, while the *practice* of New Basics attempts to address criticisms of the national curriculum movement as a top-down approach controlled from Canberra.

2.2 The Rich Tasks

The Rich Tasks are "sets of activities that students are expected to complete at specified junctures" (Education Queensland, 2000, p. 114). The juncture years are Years 3, 6 and 9, when students are assessed for their performance on the tasks. For 2001-3 and 2002-4, there are 20 tasks in the suite.

Descriptions

A brief description of the suite of Rich Tasks appears in Table 2.1.

Table 2.1

Brief description of the Rich Tasks

Number	**Task title**	**Task descriptions**: Students will…
	YEARS 1-3	
1	Webpage design	…collect information about themselves, their school and community. Design webpages. Respond to questions electronically.
2	Multimedia presentation on an Endangered Animal or plant	…investigate a threatened Australian plant or animal. Undertake constructive action. Create a persuasive and informative multimedia presentation.
3	Physical fitness	…memorise, rehearse and master dances of all forms. Investigate the role of dance and cultural context of their dances. Measure and monitor fitness.
4	Read and Talk about Stories	…view, read and listen to fiction stories presented in different media forms. Analyse characters and settings. Present ideas in a performance.
5	Historical and Social Aspects of Craft	…explore craft as a personal, social and cultural endeavour. Prepare and run a stall to showcase a chosen craft and an object made as an example.
	YEARS 4-6	

1	Travel itineraries	…design alternative itineraries of interest to a party comprising the student and an exchange student accompanied by an adult. Identify a range of issues. Present costings and reasons for their choices.
2	Narrative Text: Away with Words	…critically examine books written for emergent readers. Determine criteria for categorising these books, select one category for further examination and present a review of a book Choose an aspect of nature and create an illustrated storybook for peer review.
3	Personal Health Plan	…identify and understand an aspect of their health and fitness Develop and implement a plan to improve this aspect. Evaluate achievement of this goal and the factors contributing to the improvement.
4	A Celebratory, Festive or Artistic Event or Performance	…work within teams in planning, organising, creating and performing in an …event or performance at or outside the school.
5	Oral Histories and Diverse and Changing Lifestyles	…explore change in and diversity of 20th century lifestyles with particular reference to nature of work by recording oral histories from their own community. Present a finessed electronic media presentation.
6	Design, Make and Display a Product	…design, or improve design of, a purposeful product. Make the product or working model/prototype. Arrange public display to promote product. Flesh out marketing plan and explore suitability of materials for mass manufacture.
7	Space Futures	…engage with the exploration of space and with the techniques and procedures of the mathematical and physical sciences. Produce a model of the solar system. Iinvestigate impact of space travel on life on earth and produce design for an experiment to be performed on a spacecraft.
YEARS 7-9		
1	Science and Ethics Confer	…identify, explore and make judgments on a biotechnological process to which there are ethical dimensions. Identify scientific techniques used and research frameworks of ethical principles. Use this information to prepare pre-conference materials for an international conference featuring selected speakers who are leading lights in their chosen fields.
2	Improving Wellbeing in the Community	…work with a local community to develop a plan to improve an aspect of the community's wellbeing. Enact plan, evaluate level of success and recommend future action.
3	The Built Environment: Designing a Structure	…identify a client's needs…prepare design brief for a an environmentally sensitive and aesthetic structure. Communicate design through sketches, plans and models giving due consideration to structure, materials, quantities and costs.
4	Australian National Identity: Influences and Perspectives	…create, produce and present a powerful filmed documentary incorporating information from research and interviews with people from different cultural backgrounds to demonstrate knowledge and understanding of the different influences on Australian national identity.
5	Personal Career Development Plan	…undertake a career planning process, describe features of a range of work options and associated expectations. Assess own strengths and interests. Identify potential careers and produce individual career development plan including up-to-date resume.
6	Opinion-making Oracy	…make forceful speeches on an issur of international or national significance to three unlike audiences.
7	Pi in the Sky*	…demonstrate understanding of different mathematical approaches used to frame and answer questions about astronomy from three different cultures. Present a lesson.
8	International Trade	…identify and present detailed analysis of an export opportunity. Present a talk on this opportunity in a language other than English.

+These are described in the authors' own words and are not the published synopses for the Rich Tasks.
*The alternative to this task is *Shape We Are In*. It was made available to schools after the trial had commenced.

Each Rich Task can be mapped back to the following two aspects:

1. *Repertoires of practices* - the cognitive and cultural, linguistic and social skills needed to be acquired developmentally in order to complete the Rich Task.

2. *Operational fields* - the disciplinary, key learning areas or transdisciplinary fields of knowledge that will have to be brought into play by the student in order to complete the Task. (Education Queensland, 2000, pp. 114-115)

Consistent with the theoretical underpinnings of New Basics, the Rich Tasks are multi-faceted, multi-dimensional tasks. Each Rich Task is a culminating performance of a problem-based project that has intellectual depth and breadth, and real world value and use (Education Queensland, 2000, p. 114). It is designed to engage students with problems and issues that are substantive, meaningful and relevant to the students or their community and which extend students beyond the intellectual boundaries presented by the traditional disciplines. In order to successfully complete a Rich Task, students need to have developed skills, knowledge and processes that transcend a number of disciplines and fields of knowledge and span a range of practices. Rich Tasks call for the kind of teaching that is different from the traditional integrated, inter-disciplinary approach. Focus on a central issue or problem is expected to result in a curriculum which is less crowded and diffused, and give students more intellectually satisfying experiences. There is, however, the danger that the problem focus might lead to students missing certain topics that constitute essential and foundational learning for a particular field of knowledge. (More on this issue in another section of this report). There is the added danger that acquisition of the repertoires of practice necessary to tackle the Rich Tasks might not follow the developmental pathway that is most beneficial for that field of knowledge.

Structure

All Rich Tasks follow the same basic structure. Each Rich Task has an identifier and is presented as a very brief written synopsis. This synopsis is accompanied by complementary information in diagrammatic and written form. The written information covers how each Rich Task relates to New Basics and Repertoires of practice, and provides suggestions for strategies, the conditions under which each Tasks should be completed and indications of difficulty. Therefore, the description of each Rich Task contains the following.

(1) A *flow chart* – a diagram showing the relationship between various sub-tasks and how they result in the final product.

(2) *New Basics Referents* – a section relating the Task to one or more of the four New Basics clusters (life pathways and social futures, multiliteracies and communication media, active citizenship, and environments and technologies).

(3) *Targeted Repertoires of Practice* – a section identifying, in macro terms, the practices that students will need to call on to complete the task. There remains a great deal of discretion in the interpretation and application of each practice so that it can be applied at different standards.

(4) *Ideas, Hints and Comments* – a section suggesting possible strategies to weave the Rich Task into the curriculum plan. There are also ideas on ways of enriching teaching, learning and assessment practices.

(5) *Task Parameters* – a section setting out the conditions under the task is to be done: individually, in small or large groups; in class time or otherwise; how the task is to be presented.

(6) *Task intensity* – a section indicating the time, efforts, concentration and perseverance ("blood, sweat and tears") required of students. The fact that all Rich Tasks have medium or high intensity reflects the principle that Rich Tasks are to be substantive tasks.

(7) *Desirable features* – a section describing what is required of a high performance, and also what is present in a performance that represents an acceptable standard, that is, successful completion of the task.

Assessment

Schools are expected to spend about 40% of class time on the teaching and assessing of Rich Tasks. Formal reporting (at the time of writing, no public announcement had been about whether formal reporting would be centrally-based – i.e. on certificates issued by Education Queensland – or school-based) occurs at the end of the juncture years (Years 3, 6 and 9).

Assessment of student performance is standards-based. Student demonstrations are matched to the Desirable Features and the standards identified in the Grading Master for the Task. As described above, the *Desirable*

features section in each Rich Task shows what is required of a high performance, and also what is present in a performance that represents an acceptable standard, that is, successful completion of the task.

Student performance on the Rich Tasks is assessed by the teachers themselves but the final grade, which is reported on an official certificate, is externally moderated. The architects of New Basics see this as essential to ensure that the certificated results – and, by extension, the New Basics movement itself - have high credibility with the educational community and the public (Assessment and New Basics Branch, 2002). While the moderation model used for the Rich Tasks is being trialled with the Phase 1 schools in 2003, it is essentially an expert model with the experts (moderators) selected by the Assessment and New Basics Branch for their expertise and experience in assessment and the fields of knowledge assessed by the Rich Tasks.

Section 3:
Mathematics ideas for Years 1 to 9

The focus of mathematics teaching has been an issue in mathematics education since the 1960s. First, there was the New Mathematics of the 1960s with its emphasis on mathematical logic (e.g. set theory), the Back to Basics movement of the 1970s emphasising facts and procedures, and the Problem Solving era of the 1980s with its emphasis on adding higher intellectual functioning to mathematics syllabi. Throughout the 1990s, the debate returned to content, with increased emphasis being given to space activities (with the focus on location, direction and transformation added to shape), chance and data (increasing the importance of mathematics that deals more directly with everyday life) and algebra (where the secondary-school difficulties with algebra are being ameliorated by adding activities that enhance algebraic thinking into the primary school).

Along with these changes in content have come changes in approach driven by technological change (the advent of cheap calculators and computers) and new learning theories (particularly social constructivism, authentic learning and ethnomathematics). These have resulted in attention being given to representations (or in semiotic terms, registers), connections, communication and proof along with continued attention to problem solving. They have also prompted a rethinking of what arithmetic is now essential for students. The result of this is the move from traditional pen-and-paper algorithms to mental algorithms and the integration of calculators and computers into mathematics teaching.

These changes were originally raised in journal articles and the proceedings of international conferences such as those run by the International Group for the Psychology of Mathematics Education and the International Congress of Mathematics Education. Then they appeared in Year books (National Council of Teachers of Mathematics), handbooks such as Grouws (1992) and English (2002), and textbooks such as Booker (1997), Bobis (1999) and Steen (1997). Finally, they are appearing in declarations (DETYA, 1999), guidelines (e.g., National Council of Teachers of Mathematics, 2000) and syllabi (e.g., new 1-10 Mathematics syllabi in the United Kingdom, Queensland and NSW).

3.1 Dimensions, principles, content and processes

Because of the ease of global communication, there has been convergence within the mathematics education community as to an appropriate syllabus in mathematics. This convergence can be best seen in: (a) Alexander and Murphy's (1998) dimensions and principles (given in Table 3.1) which summarise the research with respect to the most effective ways for learning mathematics; (b) the National Council of Teachers of Mathematics (2000) principles and content and process standards (given in Table 3.2) which summarise the research with respect to what should be in a mathematics syllabus; and (c) Perry and Docket's (2000) powerful mathematics ideas (given in Table 3.3) which summarise the research on higher-order processes that act across mathematics topics.

Table 3.1

Alexander and Murphy's (1998) dimensions and principles

DIMENSIONS OF LEARNING	LEARNER-CENTRED PRINCIPLES
Knowledge base	One's existing knowledge serves as the foundation of all future learning guided organisation & representation, by serving as a basis of association with new information, and by colouring and filtering all new experiences.
Situation/context	Learning is much a socially-shared knowledge as it is an individually-constructed enterprise.
Development & Individual differences	Learning, while ultimately a unique adventure for all, progresses through various common stages of development influenced by both inherited and experiential/environmental factors.
Strategic processing	The ability to reflect upon & regulate one's thoughts and behaviour is essential to learning & development.

Motivation & affect	Motivational or affective factors, such as intrinsic motivation, attributions for learning, and personal goals, along with motivational characteristics of learning tasks, play a significant role in the learning process.

Table 3.2

National Council of Teachers of Mathematics (2000, p. 3) principles and content and processes standards:

Principles	**Content Standards**	**Processes Standards**
Equity (high expectations and strong support for all students)	*Number* (Number and operations)	*Problem solving*
Curriculum (a coherent mathematics program)	*Space* (Geometry)	*CONNECTIONS*
Teaching (understanding what students know and need to learn and challenging and supporting their learning)	*Measurement*	*Reasoning and proof*
Learning (learning with understanding by actively building new knowledge from experience and prior knowledge)	*Chance and Data* (Data analysis and probability)	*Communication*
Assessment (useful information for teachers and students to support mathematics learning)	*Algebra*	*Representation*
Technology (enhancing students' learning).		

Table 3.3

Perry and Docket's (2002) powerful mathematics ideas

Ideas (Higher-order Processes)	**Description**
Mathematisation	Process of generating mathematics from real situations and using mathematics to solve them.
Connections	Drawing connections between external and internal representations and between different mathematics topics.
Argumentation	Communication, justification and negotiation.
Number sense and mental computation	Number and operations from a perspective of understanding and flexibility.
Algebraic reasoning	Generalisation of arithmetic and the foundations of algebra.
Spatial and geometric reasoning Data and probability sense	Cover the topics of space and shape, data handling and representation (graphs) and chance, again from a perspective of understanding and flexibility.

There is also general agreement that the mathematics knowledge base constructed through teaching, learning and assessment should be *structural knowledge*, the most powerful form of mathematical knowledge. According to Sfard (1991), structural knowledge is acquired through interiorisation (a focus on the surface features in a problem context), condensation (a recognition of the underlying mathematical structure in a problem and between problems), and reification (an understanding of mathematics at an *abstract* level). Instruction to acquire structural knowledge should be based on knowledge *representations* and *connections*. Representations may be external (concrete, pictorial, diagrammatic, written symbols, spoken words) or internal (perceptions of external representations. Sfard claimed that "representations are not static, but change dramatically as learning, concept attainments and problem solving proceed" (p. 58). Halford (1994) claimed that learning involved structural connections between internal and external representations.

Mathematics consists of things, relations between things, and transformations of things (Scandura, 1971) where each relation has can be considered as a transformation and vice versa. Construction of powerful abstract understanding of relations and transformations requires knowledge that has three characteristics (Krutetski, 1978); it is (a) *generalised* (e.g., sharing between more people gives each person less – the inverse relation principle), (b) *flexible* (e.g., 61 can be 6 tens and 1 one, 11 more than half a hundred, and 1 hour and 1 minute) and (c) *reversible* (e.g., construct 2/3 from 1 whole and construct 1 whole from 2/3). Current pedagogical beliefs emphasise that the abstraction of concepts and processes is best served by a

combination of work with appropriate manipulatives and reflection with peers and teacher (English & Halford, 1995).

3.2 Framework of mathematics ideas

The focus for mathematics teaching, learning and assessment has variously been referred to above as principles, content, processes and ideas. A reading of any mathematics textbook will show other words are also used: concepts, skills, facts, strategies and procedures. Some of these words have a diversity of meanings. For example, "skills" can refer to low-level procedures such as adding 2-digit numbers, or to high-level intellectual functioning such as problem solving. As well, skills can refer to large ideas (e.g., skill in logical thinking) or to small ideas (e.g., skill in adding two-digit numbers without renaming).

In this chapter, we use the term ideas in our descriptions of the mathematics in Years 1 to 3, Years 4 to 6 and Years 7 to 9. These ideas will be oriented to processes such as communicating and to content such as common fractions.

The process ideas are based on the process standards of National Council of Mathematics (2000) modified to take account of Docket and Perry (2000) and the work on structural knowledge (e.g., Sfard, 1991). The sense and reasoning processes of Docket and Perry are not included as they form part of the content ideas that follow the process ideas. The process ideas apply across the syllabus and are as follows.

(1) *Connection* – ability to use and connect a variety of mathematics content ideas and representations (e.g., symbols, words, pictures).

(2) *Argumentation* – ability to reason mathematically, communicate and share mathematics ideas, and argue for, negotiate and justify a position (includes proof).

(3) *Mathematization* – ability to find and pose problems, generate mathematics from real situations, and use mathematics to solve problems.

(4) *Abstraction* – ability to work with mathematics in an abstract, general, flexible and reversible form.

(5) *Strategic* – ability to move beyond content, use a variety of strategies, and to reflect upon and regulate thinking and behaviour (metacogniton).

The mathematics content ideas are also based on the content standards of National Council of Mathematics (2000). These content standards were generated from over ten years of discussion and argument within the Council. They provide headings for mathematics content ideas that are also commonly used across the world (except for prealgebra which is a new idea being accepted widely in the last three years). They are as follows.

(1) *Number* – <u>concepts</u> such as whole numbers, rational numbers (fractions & decimals), integers, and rate, ratio & percent; <u>processes or content processes</u> such as identifying, representing, counting etc.

(2) *Operations* – addition, subtraction, multiplication & division, concepts, basic facts, mental and written computation, estimation, and calculator & computer methods

(3) *Prealgebra* – patterns (pattern rules), change/function, and equivalence (equals and equations).

(4) *Space* - shape (3D shape, 2D shape, line & angle,), location/direction (position & direction), and symmetry/transformations (symmetry & transformations)),

(5) *Measurement* –length, area, volume (liquid & solid), mass, temperature, time, and money

(6) *Chance and data* - probability, data handling and data display (lists, tables & graphs)

The content ideas are stated in content and not outcome terms for brevity. It will be assumed that each of these ideas has associated concepts, processes, principles, strategies, skills (low-level), procedures and facts, and can be applied to solve problem situations.

Because the Rich Tasks operate in Queensland, the mathematical ideas have been checked with the latest version of the new Queensland Year 1-10 Mathematics syllabus. However, their overall structure has come from a more general analysis of mathematics required for school students.

3.3 A Years 1-9 mathematics "summary syllabus"

The content ideas have been organised into a "summary syllabus" based on Years 1-3, 4-6 and 7-9 (see Table 3.4). This "summary syllabus" is based on syllabi developed in Australia, England and the US National Council of Mathematics.

Table 3.4

Mathematics "summary syllabus" for Years 1-3. 4-6 and 7-9

IDEA	YEARS 1-3	YEARS 4-6	YEARS 7-9
		NUMBER	
Whole numbers	Numeration to 999 – Reading, recording, & representing numbers – Counting and place value – Seriation, comparing and ordering – Renaming and regrouping – Approximating and estimating – Ordinal number ▪ pplications to money and measurement – Reading and recording dollars and cents – Reading and recording m/cm	Numeration to 999 999 – As for Years 1-3 – Multiplicative relationship between place-value positions ▪ rimes, composites, roots, polygonal numbers ▪ pplications to money and measurements – Cash/cashless transactions – Reading , recording metric measures that are related by 1000	Numeration beyond millions – As for Years 4-6 – Whole number system Integers (positive, negative) Squares, cubes and roots Index notation ▪ pplications to money and measurements
Common fractions	▪ dentifying equal parts of a whole (area model only) Naming unit fractions by linking to ordinal number names (to tenths) Identifying unit fractions to tenths ▪ nformal recording (e.g., 1 fifth, 1 tenth) ▪ artitioning a whole to represent unit fractions ▪ onstructing a whole from a given part	▪ s for Years 1-3 including number line and division models (non-unit fractions; mixed numbers) – Reading, recording (formal), representing – Counting, comparing and ordering – Renaming and regrouping – Approximating and estimating	Common fractions – Part of a group and multiplier concepts – Mixed numbers ↔ improper fractions – Equivalent fractions
Decimal numbers	▪ enths (as part of fraction concept development)	Tenths, hundredths, thousandths Processes as for Years 4-6 common fractions (including "mixed numbers" e.g., 3.356) Extending multiplicative	As for Years 4-6 Decimal number system ▪ pplications to money and

		relationships to decimals Limited applications to money and measurements	measurements – Best buys/budgets – Interest/discounts Rational number system
Percent, rate and ratio		Percent – Concept (hundredths) – Simple percent applications (e.g., discount, profit)	Percent – Further applications (e.g. interest) – Difference between percent, percentage & amount (100%) Rate and ratio – Concepts and differences between them – Equivalent ratio (proportion) – Applications (e.g., scale)

OPERATIONS			
Add/Subtraction			
Concepts	Addition as joining, inverse of subtraction, and comparison Subtraction as take-away, inverse of addition (missing addend), and comparison ▪ onnections between addition and subtraction	▪ ddition and subtraction as part-part-total	As for Years 4-6
Principles	Difference, commutative, inverse	▪ s for Years 1-3, inverse relationship, associative	As for Years 4-6
Basic facts	▪ trategies and recall to 9+9 (and 18-9) Extensions to larger numbers	As for Years 1-3	As for Years 4-6
Computation	<100: mental and written (idiosyncratic, algorithmic) ▪ 100: calculators, computers, estimation	≤ 1000, simple common and decimal fractions: mental and written >1000: calculator, computer, estimation ▪ pplication to money and measurement	As for Years 4-6 ▪ ecimal and common fractions: calculator; computer, estimation ▪ pplication to measurement formulae (e.g., Perimeter)
Mult/Division			
Concepts	Multiplication: equal groups→array model Division: partition (equal shares)→quotition (equal groups)	Multiplication as inverse of division, multiplicative comparison and combinations Division as inverse of multiplication, multiplicative comparison and combinations Multiplication and division as part-part-product	Multiplication as part of a part Division as multiplication by inverse
Principles	▪ nverse, direct & inverse relationship (informal)	▪ ommutative, associative; inverse, inverse relationship	As for Years 4-6; distributive
Basic facts	Basic facts to 9 × 9	Basic facts to 81 ÷ 9	As for Years 4-6

Computation	<100: mental and written (idiosyncratic, algorithmic) − limited to 1-digit multipliers/divisors >100: calculators, computers, estimation	▪ xtensions of strategies to larger numbers ≤ 1000, simple decimal fractions: mental and written >1000: calculator, computer, estimation Application to money and measurement	≤ 1000, simple common and decimal fractions: mental and written (1 and 2-digit multipliers/divisors) >1000: calculator, computer, estimation Decimal and common fractions: calculator, computer, estimation Simple percents: Mental and written ▪ ther percents: Calculator, computer, estimations Rate, ratio, proportion: Students to choose computational method, calculator, computer Application to financial planning & measurement formulae (e.g., area)

EARLY ALGEBRA			
Patterns	▪ ttribute discrimination, sorting and classifying Pattern rules to +/- – Patterns with one or more attribute changes including +, – – Rules based on repeating parts and relation to previous term	Pattern rules to x/÷ Patterns based on x/÷ – multiplication and division – Rules based on position	Pattern rules to variable – Represent pattern rules as generalities to introduce variable
Change/ functions	Change rules to +/- – Input-output function machines for one or more attribute Using rules between sets of data to predict output and backtrack to input	Change rules to x/÷ – Input-output function machines – Reinterpret arithmetic in terms of change (use arrow diagrams) – Use rules between sets of data to predict output and backtrack to input Apply rules to graphs to determine relationships between two quantities	▪ hange rules to variable and function – Represent change rules (relationships) as generalities to introduce variable and as ordered pairs to introduce function – Represent generalities/functions with graphs (linear and non-linear) – Use computers to represent general relationships/functions ▪ olve linear equations using backtracking
Equivalence Equals	Concepts of equals and not equals	Balance – Concepts of greater and smaller – Use guess and check and balance with respect to =	Balance – Use relationships such as formulae to introduce the notion of variable – Relate balance to two-way change (backtracking) as concepts of equals and equivalence
Equations	Representations to +/- – Symbols for equals and not equals – Using equals/not equals in equations Using spaces and boxes to find unknowns in simple addition and subtraction equations	Representations to x/÷ – Symbols for greater than and less than – Use equals/not equals in equations Use spaces or boxes and balance to find unknowns in multiplication and division equations	Representations to variable – Use letters to represent variables in expressions, equations and functions and relate these to pattern rules, change rules, formulae and arrow arithmetic – Use variables and guess

			and check, balance or backtracking to solve for an unknown – Interrelate variable expressions, equations and functions to ordered pairs, tables and graphs Familiarise with computer representations of generalisation, variables and equations

SPACE			
Shape *3D shape*	Common solid shapes (cube, sphere, pyramid, cylinder) – real-world; abstract Properties (flat, curved, round, pointed, faces/sides, corners, & edges) Relationships between: solid/plane shapes; packaging shape and contents	Formal language (e.g., vertex, octahedron) Classifications of shapes by properties (e.g., all prisms have 2 congruent and parallel faces/bases, and the remaining faces are rectangular) Relationship between prisms and cylinders	Systems of shapes (e.g., polyhedra, platonic solids) Relationships between solid shapes and nets, and vertices, edges and faces (e.g., Euler's formulae) Cross sections
2D shape	Common plane shapes (square, rectangle, triangle, circle), informal polygons ▪ roperties (straight, curved edges) Boundary (i.e., perimeter) Enclosed region (i.e., area)	▪ ormal language (e.g., trapezium) Classifications of shapes by properties (e.g., quadrilaterals, polygons) Relationships between plane shapes and line and angle properties.	▪ ystems of shapes (e.g., polyhedra) Systematic study of rigidity, diagonals and angle properties
Line	▪ traight, curved, continuous, 1-dimensional	▪ arallel, intersecting, perpendicular	As for Years 4-6
Angle	▪ oncept (amount of turn); right angle	Acute, obtuse, straight angle ▪ nterior, exterior angles (plane shapes)	Reflex, corresponding, vertically opposite, alternate angles ▪ elationship between interior angles/plane shapes ▪ ythagoras & early trigonometry
Location/ Direction Position	Language and simple grids Left, right, up, down; east, west, north, south 1st, 2nd	Cartesian and polar coordinates Maps	Coordinate systems (Cartesian & polar) – Scale drawings – Plotting position (world & space)
Direction	Language Following & giving directions in terms of steps & part turns Spatial orientation	▪ ompass points ▪ levations ▪ ooking at & drawing objects from different perspectives	Compass bearings ▪ uilding plans & perspective drawings ▪ lanning travel & plotting movement (world & space)

			▪
Symmetry/ Transformations Symmetry	Concepts of line and rotational symmetry	▪ lassification of shapes by numbers of line and rotational symmetries ▪	Mental rotation Relationships between line and rotational symmetry Modifying and constructing symmetrical shapes
Transformations	Simple dissections (e.g., tangrams) and tessellations	▪ oncepts and properties of slides, flips and turns ▪ elationships between slides, flips, turns Properties of tessellating shapes Construction of dissections and tessellations, and applications in design	▪ rojections (reductions, enlargements) ▪ elation of transformations to dissections Congruence, similarity and relationships to transformations and shapes

MEASUREMENT			
Length/ Perimeter	Metre, centimetre ▪ oncept, language, direct/indirect comparison, Non-standard → standard units. formal symbols ▪ onversions (informal) e.g., 157 cm↔ 1 m 57 cm Concept of perimeter as distance around	Millimetre, kilometre Formal standard units and symbols ▪ umber system relationships (ones, thousands) ▪ ormal conversions (km, m, cm, mm) Concept of perimeter and relationship between perimeter and length of sides	As for Years 4-6 Perimeter: Relating perimeter to sides and circumference to radius or diameter
Area	Area (non-standard units only) ▪ oncept, language, direct/indirect comparison, Non-standard → standard units	▪ quare metre, square centimetre, hectare ▪ ecording (informal for sq, cm, sq.m) Relationships: between volume, area, length, width of rectangle; between triangle and rectangle that encloses it; between area and radius (informal)	Square kilometer ▪ ormal recording (m^2, cm^2, km^2) Formal conversions Calculating area of rectangle, triangle, circle, and irregular shapes
Volume Liquid Solid	Litre ▪ oncept, language, direct/indirect comparison, Non-standard → standard units ▪ oncept, language, direct/indirect comparison, non-standard units of measure	Millilitre, kilolitre Formal symbols (e.g., mL, kL) Formal conversions (mL, L) Cubic metre, cubic centimetre Informal recording (e.g., cu. m) Number system relationships (ones, millions; ones, ten-thousands) Relationships between volume, area of base rectangular prism and height	As for Years 4-6 Formal recording (m^3,cm^3) Conversions between units ▪ elationships between area of base and volume Calculating volume of prisms Measuring techniques (e.g., for volume of irregular solids)
Mass	Kilogram ▪	Gram, tonne ▪	As for Years 4-6 Applications to problems

	oncept, language, direct/indirect comparison, Non-standard → standard units	ormal standard units and symbols (e.g., gram, g) Formal conversions (t, Kg, g)	
Temperature	Degrees Celsius (positive) Concept, language and direct/indirect comparison Standard units (degrees Celsius) − + integers only	As for Years 1-3 ▪ easuring temperature (positive)	Degree Celsius (negative); boiling./freezing points; weather, antifreeze ▪ easuring temperature (negative)
Time Concept Telling time	Hour, day, week, month, minute, season, year, calendar; relations/conversions between units Concept, language, direct/indirect comparison, non-standard units and standard units ▪ 'clock; half past, quarter past (analogue, digital)	Second, leap years, decade, century, a.m./p.m., timetables; diaries ; clocks ▪ onventions for dates (long, abbreviated) Simple time computations - mental/estimation Minutes past, quarter past/to (analogue, digital)	▪ ime to decimal representation (2.5 hours) ▪ ime zones, daylight saving, BC/AD timelines Calculating time intervals, starting and/or finishing times ▪ inutes to; stopwatches; 24-hour time
Money	Dollars and cents ▪ oncept, language, direct/indirect comparison (barter), non-standard & standard units	Large denomination bills Computation with money Currencies around the world	Financial mathematics – Budgeting, planning, financial management – Best buys, credit, interest – (computer, calculator)

CHANCE AND DATA			
Chance Concept	One sample space – Identifying events that are certain, possible or impossible – Language – e.g., always, sometimes, never, some chance, no chance, every chance	▪ wo or three sample spaces with an equal number of outcomes in each (enables whole-number comparison) – Fairness and outcomes – Trial, experiment; likely, unlikely, less, more, most, equally likely; equal chance	▪ heoretical probability based on equally likely outcomes (requiring fraction comparison knowledge) ▪ xperimental probability based

Outcomes	Comparing outcomes in chance events and their relation to previous events	– Comparing ▪ ikelihood of outcomes of a chance event in a single sample space (e.g., most likely) ▪ ikelihood of outcomes when events are consecutive, when the consecutive nature of the events changes and does not change the event (independence of events) ▪ electing the event more/most likely to give required outcome in 1, 2, and 3 sample spaces Calculation – Relating chance to fractions between 0 (impossible) and 1 (certain) – Working out chance of outcomes from experiments	on trials ▪ ists, frequency tables, and tree diagrams Calculation – Calculating theoretical probabilities – Using experiments and computer simulations to determine experimental probabilities and relating this to theoretical probabilities Application – Using probability to make predictions about events – Analysing events for fairness/bias
Data Handling	Pictures List and tables (tallying)	▪ ounding data ▪ urveys, observations and experiments ▪ ▪	▪ nvestigations involving a variety of data methods Forms of data (discrete, continuous)
Display	Simple picture and bar graphs ▪ sing picture and bar graphs to compare data	Many to one picture graphs Complex bar graphs, line graphs, axes, scales and titles Pie charts and dot plots Tables and charts (e.g., bus timetables)	▪ ompound graphs, stem & leaf plots, histograms

	▪ imple tables (2 column)	Use displays to compare and to determine trends and relationships Use of computers	▪ ompound tables and charts (including flow charts, timelines, etc.) Measures of central tendency (mean. mode and median) Use displays to draw inferences and make predictions

Section 4:
Issues affecting the integration of mathematics and Rich Tasks

The key concern of this report was to find the answer to the question: *How effective are the Rich Tasks as a vehicle for teaching mathematics in Queensland schools?* In Sections 2 and 3, we have provided the context of Rich Tasks and a framework for mathematics. Putting these two together gives rise to issues that affect the integration of mathematics and Rich Tasks.

As part of our analysis of the Tasks, we interviewed schools involved in trialling the Rich Tasks. Two schools (out of six) agreed to the interviews and produced two case studies to illustrate the dichotomy that could result from different ways of implementing the Rich Tasks. The two case studies came from the Middle School part of a metropolitan P-12 school (School A) and a country secondary school (School B).

4.1 Issues

Issue 1: Rich Tasks and mathematics content ideas

Each Rich Task has a section called *Targeted repertoires of practice* which lists those skills that students need to acquire in order to complete the tasks. These skills cover the cognitive, cultural, linguistic and social domains. The repertoires are identified in macro terms and do not list the precise concepts, techniques and procedures that are necessary.

However, a strong focus on problems and issues is a defining characteristic of the Rich Tasks. The Tasks have relevance and significance in today's world and call for teaching and learning that is transdisciplinary. Thus, there is a very strong emphasis in the Rich Tasks on *investigation* (e.g., data handling, hypothesis formulation and testing and drawing inferences – which can involve chance). In particular, the nature of the Rich Tasks appears to have a major focus on *presentation* in some form. It is likely that mathematics content that relates to presentation (e.g., whole numbers, following patterns, shape and location/direction, data display and interpretation (in the form of graphs and tables), and measurement of length, mass, temperature and time) will predominate in Rich Tasks.

While these are substantive and substantial intellectual skills, there exists the possibility that certain major mathematical concepts might be overlooked if they do not fit the requirements of the Tasks, either as embedded or prerequisite mathematics. This is acknowledged by the Assessment and New Basics Branch (Assessment and New Basics Branch b, 2000) in one of its publications.

> As a set, the New Basics clusters are not meant to be exhaustive, nor could they be. They embrace selections from an infinite set of possibilities. Teachers and schools will include other knowledges – local, classical, multicultural and alternative. (p. 6)

As well, there is the added possibility that the concepts might not be taught in the most beneficial sequence.

Issue 2: Rich Tasks and mathematics process ideas

The Rich Tasks offer students the opportunity of investigating life-like situations. These situations may involve the students working in groups. Group work means that the students have to talk with each other and to share their ideas, develop plans for the investigation and for the presentation. This will involve the students reasoning about what to do, negotiating, arguing for and justifying their position. This communication, negotiation and justification is the *argumentation* process idea.

The Rich Tasks are very open and leave a lot of options for the students. This means that the students have to find and pose problems, look for the mathematics in the problems and then use this mathematics to solve these problems. This is the *mathematisation* process idea.

Our research experience has shown that argumentation and mathematisation are processes that are often not used in mathematics teaching. The Rich Tasks, therefore, provide an avenue for important developments in mathematics learning. However, the effectiveness of Rich Tasks for the other three mathematics process ideas, connection, abstraction and strategic, is more problematic.

For connection, the mathematics in Rich Task situations is often in context and therefore in either a physical or visual representation. However, it is not often in the form of symbolic representation and appropriate language representations may not be used. However, the Tasks do set up situations where representations and connections between representations can be considered. As well, the Tasks require, similar to all problem situations, students to connect mathematics ideas so as to come to necessary conclusions. Thus, Rich Tasks provide a situation for considering connections but any connections used would have to be developed before the Tasks.

For abstraction, there is a real difficulty for Rich Tasks. This is discussed in Issue 3.

For strategic, solving the problems and completing the Tasks involves going beyond mathematics content to using of strategies and metacognition, particularly reflection and regulation of thinking. This is the *strategic process* idea. However, the Rich Tasks only provide an avenue for the application of strategies and metacognition – the strategies and the metacognitive abilities must be learnt prior to the Tasks.

Issue 3: Authenticity versus abstractness

New Basics was born of a strong commitment by its architects to deliver a schooling experience that has "visibility of purpose – ensuring discipline rigour and contexts for learning that are connected to the world" (Assessment & New Basics Branch a, 2000, p. 4). The authenticity focus is the underpinning principle of Rich Tasks, and its architects argue that relevant work is one of the two characteristics necessary for improved student outcomes (Assessment & New Basics Branch b, 2000).

For skills to have life-long educational value, they must be able to be transferred from one context to another. It is the transferability of a skill that makes it a true basic skill (Pitman, Matters and Nuyen, 1998), and transferability requires the ability to abstract the concept from its applications. This is an issue that teachers of Rich Tasks need to deal with.

The Rich Tasks can often be successfully completed by only using low-level mathematics. For example, whilst money and metric measures are based on decimal fractions, they can often be successfully handled as whole numbers. As well, understanding of underlying mathematics is often unnecessary for completing the Rich Tasks – rote procedures can suffice. Therefore, there is the danger that the Rich Tasks will be undertaken at the procedural level in order "to get the job done" and that the mathematics experienced by students in solving the Rich Tasks could remain contextual and applied.

It is the abstract form of mathematics (the structural mathematics of Sfard, 1993) that provides its <u>power and transferability</u>. According to Ohlsson (1993), a particularly powerful form of structural knowledge is the abstract schema that encode for structure rather than content. Abstract schema are more likely to connect with new representations than are impoverished networks because there is more to which they can relate. The structure of rich connected internal networks of knowledge enhances the student's ability to search for similarities and differences between tasks and has more potential for transfer (Hiebert & Carpenter, 1992).

Thus, to ensure that structural knowledge is developed and reinforced by the Rich Tasks, teachers will need a willingness to ensure that the higher-level outcomes are covered. As well, the teachers will have to have excellent subject matter knowledge to identify the underlying abstractions of mathematics used in Rich Tasks.

Issue 4: "Bolt-on", "built-in" and engagement

We have borrowed here two terms from Pitman, Matters and Nuyen (1998) to describe two different situations based on two very different approaches to the implementation of New Basics in schools. In the "bolt-on" approach, teachers feel that the reform agenda is forced on them and they see the Rich Tasks as extras on top of an already crowded curriculum. This situation is particularly relevant to mathematics, the subject that is often seen by teachers as the most crowded of all curricula but so important that nothing can be left out. Therefore, if teachers cannot easily see ways that mathematics can be integrated with the Rich Tasks, they will resent the Tasks. In the "built-in" model, the Rich Tasks are regarded as an intrinsic and natural component of the curriculum experienced by students, and teachers work together to find ways of weaving them into the curriculum.

New Basics is an educational reform that involves more than curricular and pedagogical reform: it is fundamentally about a different way of providing students with powerful and meaningful experiences. Teachers need to work collaboratively across disciplines in order to achieve the desired outcomes for

students. If Rich Tasks have only been "bolted on", then there may not be the level of engagement with the Tasks by the teacher that is required to provide motivation for the extra work that is required for productive collaboration.

But, more than that, both teachers and students need to venture outside the school environment in order to complete the Rich Tasks. The Assessment & New Basics Branch, citing Sizer (Assessment & New Basics Branch b, 2000, p. 5), refers to those schools where the whole school community is engaged in the planning and presentation of Rich Tasks as "mindful schools", where the Rich Tasks serve as "a means of celebrating the intellectual life of the school" (New Basics Branch b, 2000, p. 5). Historically, mathematics is a subject that many teachers believe should be taught within classrooms in transmission form. Thus, the imperative in the Rich Tasks to out-of-class work may clash with teachers' views of mathematics teaching and learning.

Therefore, it is hardly surprising that the two different approaches ("built in" and "bolt on") deliver different engagement and, thus, different outcomes in terms of student achievement and pedagogy. This is illustrated in the two case studies that we will see in a later part of this chapter.

4.2 Case studies

From the issues above, it is quite clear that the planning and implementation of Rich Tasks will impact on the organisation of different schools in different ways, with some implementation models being less problematic than others. The two schools that we have used to illustrate our analysis represent two opposite ends of the continuum with respect to issue 4 above, leading to very different impact on student outcomes and staff morale.

School A was a New Basics Phase 1 school, with an enrolment of over 1,100 students. It is located in a middle-class suburb with many young families in a traditional family structure. The school offered classes from pre-school to Year 12, including special education in multi-age groups similar to the years of the Rich Tasks. As a consequence of this, the school has a relatively large management group. The interviews focused on the Upper Junior School and Middle School, Years 4 to 6 and 7 to 9.

School B was also a New Basics Phase 1 school, with an enrolment of about 350 students. It is located in a country town some distance from Brisbane. The town is relatively prosperous with a very low unemployment rate. The school offers classes from Year 8 to Year 12 and has about 30 feeder schools into Year 8. It had a traditional management structure, with a head of department heading each subject department.

School A, the middle school

The Middle School (Years 7-9) had completed one Rich Task in 2002 and was planning to do the remaining Tasks with Years 8 and 9 students in subsequent years while the Upper Junior School (Years 4-6) had done two Tasks whose main focus was literacy.

In the Middle School, all classes were required to complete a program focusing on higher intellectual functioning called *depth studies* and were allocated two extra lessons of 1.5 hours each as an adjunct to these depth studies. These two lessons were available for the Rich Tasks but the students needed some of these hours to complete depth studies so not all the hours were devoted to Rich Tasks. The School was currently mapping its numeracy program to align with its depth studies but the Rich Tasks were not being considered to form part of this exercise. They are considered "just another project, but of more intensity". The key issue for the School was its multi-age structure and three-year cyclic curriculum.

The teachers' responses to the interviews indicated that they found the Rich Tasks' repertoires of practice "very light in terms of numeracy" and, as a consequence, they built their own repertoires. They saw Rich Tasks as standing alone in the curriculum and they believed that they had to continue to teach mathematics predominantly outside the Rich Tasks. They admitted that they taught the Rich Tasks as "little bits here and there" and did not fit the Tasks into the mathematics teaching sequence. The primary teachers' responses, in particular, showed that they were finding it difficult to come to terms with the standards-based model. They believed that "time constraints" were such that could not adopt a standards model. They also indicated that some parents so lacked understanding of criteria and standards that they were taking their children out of the school.

School B, the country secondary school

The staff of the country school was ready for a change when the trial of New Basics was available. The school had participated in the Education Queensland/University of Queensland joint project *School Reform Longitudinal Study* (Luke et al., 1999), one of the four research studies that constitute the research premise for New Basics. The school therefore readily embraced New Basics and Rich Tasks and spent the first year of the trial on an implementation plan in which they trialled two Tasks in Year 8.

The main characteristics of school B's Year 8 implementation plan in this first year were:

- All Year 8 teachers were assigned to Rich Task teams. Each team was given two Tasks and were led by two leaders. One term was devoted to each Rich Task.
- The teams studied the Tasks and decided among them how the Tasks were to be taught. For instance, in the *Community Wellbeing* Task, the mathematics teachers taught the data collection and data analysis while the SOSE teachers taught students the concept of "community".
- The teachers in the teams, with the help of the Head of Department, rewrote the Rich Task descriptions and targeted repertoires of practice in terms that they felt their students could more easily understand. They also tried to give more structure to each Task as they found that male students had difficulty coping with open tasks.
- The year was organised so that only half of the Year 8 group would be doing a Rich Task at any one time. This was planned to avoid overloading the small local community with requests from the school.
- Parents were included in the planning through a parents' evening held at the beginning of Year 8 to explain New Basics and Rich Tasks.
- A whole-school "Adventure Program" involving all students was organised to run parallel to the teaching of the Rich Tasks. In this program, students were taught leadership and team skills to achieve life-long learning, an enriched community and flexible pathways to the future. The program's aims were self-awareness, critical reflection, personal development, communication, co-operation, application, commitment, and confidence. It was hoped that the program would provide students with training in independent problem solving and pastoral care.

School B had developed a plan for implementing the remaining Tasks in Year 9. This plan identified the mathematics that was embedded as well as the prerequisite mathematics for each Rich Task. The work program was then audited against the syllabus to ensure that no major concepts were left out. The prerequisite mathematics was then taught in either Year 8 or Year 9, depending on the topics and the time at which the Rich Task would be undertaken. Where mathematics ideas were not covered in any Rich Task, the teachers taught these ideas when the students are doing a Rich Task that did not have a lot of mathematics or when there was a break from the Tasks. An example should illustrate this point. The plan was for the students to study algebra in Year 9 in the same term they were to do *Opinion-making Oracy*, a Rich Task with little mathematics.

The teachers had identified three types of mathematics teaching: (a) "mathematics before the Rich Tasks", (b) "mathematics in the Rich Tasks", and (c) "mathematics outside the Rich Tasks". In this way, the teachers felt that everything was covered, irrespective of whether the topic was required for completion of the Rich Tasks. The teachers had not considered using Rich Tasks as motivation for mathematics (the associated mathematics of this report). When shown an example of this, they were interested and said that they would look into this in the next year.

Being secondary teachers, the teachers had no problem working with criteria and standards. They reported that their assessments of the students were almost perfectly aligned with the standards on the Grading Masters. They did however feel it necessary to rewrite the Grading Masters issued by the assessment and New Basics Branch to make them more easily understandable by students and parents.

4.3 Implications

There are two implications from the issues and the case studies; the importance of "building in" and engagement, and the need to widen the mathematics content and process ideas that are related to each Rich Task.

"Building in" and engaging

It is quite clear that the teachers of School A (including three heads of curriculum) interviewed for this report did not consider the implementation of New Basics at their school a success. Their implementation approach was obviously a "bolt-on" model where the Rich Tasks were quarantined and put on the sideline while the "main" business of the school was carried on as usual. There was no attempt to weave the Rich Tasks into their mathematics curriculum; there was no whole-school engagement in the planning and implementation of Rich Tasks.

This means that the problem of overcrowding of the mathematics curriculum was exacerbated and not resolved by the introduction of the Rich Tasks. Yet, cutting through a crowded and potentially diffuse curriculum is one of the two main raison d'être of New Basics. The other great potentially beneficial impact of Rich Tasks – to give students a richly integrated schooling experience - was also not being realised.

The implications of this are that by bolting Rich Tasks on to the school curriculum and by keeping a clear division between Rich Task work and other work, the school may fail to show students that schooling is relevant and school experiences can be both intellectually engaging and relevant to the students themselves and their community.

However, for school B, the teachers interviewed (the head of the mathematics department and two classroom teachers) felt that New Basics and Rich Tasks had been "built in" to the teaching practices of the school and had rejuvenated the intellectual life of the school. There was impact on the school organisation as well as its timetabling practices. The whole school was engaged in the exercise and used the opportunity to implement some important new initiatives. By introducing Rich-Task teams, the school was allowing more teachers to be exposed to leadership roles, an opportunity that they may not have had otherwise.

The teachers' responses indicated that they felt the impact on student outcomes had also been most positive. The school reported that the Rich Tasks served as a strong motivator for the students and student performance has improved in all subjects. Furthermore, by engaging in true transdisciplinary teaching and learning, the school has made room in the curriculum for the teaching of personal, social and life skills that would have positive impact on student performance on the Rich Tasks.

Thus, the two case studies reinforce Issue 4 that whether the school builds the Rich Tasks into the school's curriculum or bolts them on is crucial to the successful implementation of Rich Tasks.

Prerequisite, embedded and associated mathematics content ideas

It is evident from the interviews that the mathematics teachers at School B had been able to analyse the Tasks in terms of different types of mathematics and so find sufficient mathematics things to do within the Tasks. They did not feel that the Tasks were adversely affecting their ability to deliver the required mathematics ideas and saw the Tasks as being able to deliver mathematics learning in many areas (and not just low level ones). With their "mathematics before the Rich Tasks", "mathematics in the Rich Tasks", and "mathematics outside the Rich Tasks", the teachers of School B had found a way to ensure that abstract learning was catered for as well as authentic. This was particularly seen in the way they integrated algebra around the Tasks.

However, discussions with the teachers at School A showed that they had not looked at mathematics in terms of how its teaching could be motivated by the Rich Tasks. This leads to three relationships when analysing a Rich Task for its mathematics - that which is needed before the Task can begin, that which is obviously part of the Task, and that which can be motivated by the Task. An example should serve to illustrate these three relationships.

In the Rich Task *Personal Career Development Plan*, presenting complex ideas textually and graphically is central. Thus data display in the form of flowcharts and timelines is a component of the Task. This kind of mathematics was identified by School B and is referred to as <u>embedded</u> in the Task. However, flowcharts and timelines require prior knowledge such as understanding of the concept of time. This kind of mathematics was also identified by School B and is referred to as <u>prerequisite</u> to the Rich Task.

However, there is another possibility. Exploring a multiplicity of possibilities is listed as a targeted repertoire of practice for the Rich Task. This repertoire could be interpreted to encompass one, some or all the following:

- obtaining information (from reading books, researching the internet, interviewing people etc.) about the kind of jobs available in different industries and locations;
- assessing the likelihood of obtaining training for the jobs in which I have an interest;
- calculating the likely financial cost of such training; and
- assessing the chance of obtaining employment when I have completed my training.

The last three tasks require knowledge of chance or probability – a major mathematical concept. Yet, the task could be completed without reference to this concept. However, the Rich Task provides teachers with an excellent opportunity to teach probability in an authentic manner. This type of mathematics, which can be considered as being engendered by the Task, was not identified by School B. However, it provides further opportunities for using the Tasks for teaching mathematics and should be used. It is referred to as associated with the Rich Task.

Thus, this report will analyse the Rich Tasks in terms of the three types of mathematics (see Section 5):

(1) *embedded mathematics* – mathematics ideas that are clearly identified as a component of the Task requirements.

(2) *prerequisite mathematics* – mathematics ideas that are part of the prior knowledge necessary to understand the embedded mathematics.

(3) *prerequisite mathematics* – mathematics ideas that are not embedded or prerequisite but can be motivated or given authenticity by the Rich Task (i.e., the Rich Task serves as a vehicle to extend the teaching of mathematics).

The interviews with School A seemed to indicate that the "bolt-on" approach was a consequence of not seeing the Rich tasks as able to meet many of the mathematical needs of the students. It is evident that seeing mathematics within the Rich Tasks in terms of prerequisite, embedded and associated would assist the teachers.

Integration of mathematics process ideas

The brief analysis of process ideas and the Rich Tasks in Issues 2 and 3 earlier in this Section revealed that the Rich Tasks were more supportive of the argumentation and mathematisation process ideas than they were of the connection, strategic and, particularly, abstraction process ideas. Thus, what are the implications of the issues and the case studies for successful integration of Rich Tasks and process ideas such as abstraction?

The interviews indicated that School A teachers had a perception that the Tasks would only deliver low level outcomes and may not meet the need to teach mathematics at an abstract level. Contrary to this, the teachers of School B did not share School A teachers' beliefs about Rich Tasks and abstraction. In fact, interviews indicated that School B's teachers did not believe that the Rich Tasks put the development of abstraction and strategic understanding at risk. This was because of two aspects of their program. First was the school's adoption of another program that supported the investigatory nature of the Rich Tasks. The school had set up a whole-school program of social, interpersonal, leadership and survival skills. With this program and the Rich Tasks, School B teachers believed that their school was developing thinking and reasoning at a high level that supported abstract and strategic thinking. Second was the school's acceptance that some mathematics was not in any Rich Task (mathematics outside Rich Tasks), and that time had to be set aside for these ideas to be developed separately. In particular, School B realised that the most general of the mathematics ideas, algebra, needed time to be developed at the symbolic level.

Thus, this means that there has to be an integrated study of mathematics that covers:

- the Rich Tasks,
- the mathematics content and process ideas not covered in the Rich Tasks, and
- other additional programs that support both Rich Tasks and mathematics.

To this end, Section 6 will provide the mathematics content and process ideas not covered in the Rich Tasks, and a way in which all the mathematics ideas and concepts identified in this Report can be integrated in the Rich Tasks and covered in each of the three-year intervals, Years 1 to 3, Years 4 to 6 and Years 7 to 9.

Section 5:
Mathematics analysis of the Rich Tasks

The Rich Tasks provide the students with investigations that operate across traditional disciplines requiring the integration of knowledge from many of the disciplines. Some Tasks explicitly require mathematics knowledge to be used; others do not. Therefore, each Task is analysed for mathematics from the "summary syllabus" in three ways:

(i) for mathematics that is *prerequisite* for the task and should be taught before it;

(ii) for mathematics that is *embedded* in the task and can be taught by doing the task; and

(iii) for mathematics that is *associated* with the task and can be engendered by the task.

All other mathematics ideas are *outside* the Rich Tasks and have to be taught separately.

5.1 Mathematics ideas for Year 1-3 Tasks

For Years 1 to 3, the major mathematics ideas from the "summary syllabus" for the Tasks are as follows.

Task 1: Webpage design

This Task asks students to collect information about themselves and their environment and then to compose visually appealing web pages.

Mathematics is involved in the steps of *collecting the information* and *composing the webpages*. Data handling and display is obviously embedded in this Task. As well, the information collected will involve length, mass, temperature time and have whole numbers as a prerequisite. Table 5.1 summarises the mathematics in Task 1.

Table 5.1

Mathematics ideas for Year 1-3 Rich Task 1 Webpage Design

Prerequisite mathematics	Embedded mathematics	Associated mathematics
Whole numbers	Data - *handling & display* Length – *standard units* Mass – *standard units* Temperature – *standard units* Time – *standard units*	

Task 2: Multimedia presentation of an endangered plant or animal

This task involves selecting a local threatened plant or animal, researching its status, compiling a report and a class retrieval chart, and preparing a multimedia presentation.

Any mathematics will be part of the main steps of the task: *selecting/researching a plant or animal*, *compiling a report/chart*, and *preparing a multimedia presentation*. Selecting an animal or plant involves consideration of chances of extinction, looking particularly at meanings of certain, impossible and possible and comparing likelihoods of outcomes. The concept of chance would certainly have to be known. Similarly data handling and display techniques such as tables and charts will be involved in collecting and presenting information. Finally, similar to Task 1, there may be some motivation for learning location and direction in the preparation of the multimedia presentation, particularly in the maps of where animals are.

However, understanding the data gathered in order to select an animal or plant would be enhanced if the students have knowledge of whole numbers (up to 3-digit numbers and including comparing numbers),

addition and subtraction (an endangered species should have reducing numbers), and length, mass and time (an endangered species may show changes in length and mass across time). The yearly data on an endangered species is also an example of a subtractive sequence of numbers that could be considered as patterns. This is a fair amount of associated mathematics. Table 5.2 summarises the mathematics in Task 2.

Table 5.2

Mathematics ideas for Year 1-3 Rich Task 2 Multimedia Presentation

Prerequisite mathematics	Embedded mathematics	Associated mathematics
Chance - *concept* Whole numbers	Chance – *outcomes* Data – *handling & display*	Location/dir. – *location & direction* Length Mass Time Patterns

Task 3: Physical fitness

This task involves students performing dances as a physical activity, *monitoring* and *measuring* their fitness as they engage in this activity.

The main mathematics involved here is in *memorising* and *performing the dances*, keeping the *data log* and *measuring* fitness levels. Dancing is an activity involving location and directions and following movement and sound patterns. Whole numbers are a prerequisite, data handling and display are embedded and there is the opportunity to teach students about operations as they compare the levels of fitness before and after the dances. Table 5.3 summarises the mathematics in Task 3.

Table 5.3

Mathematics ideas for Year 1-3 Rich Task 3 Physical Fitness

Prerequisite mathematics	Embedded mathematics	Associated mathematics
Whole numbers	Patterns Location/dir. – *location & direction* Data - *handling & display*	Add/subtr. – *concept, basic facts & computation*

Task 4: Read and Talk about Stories

In this task, students engage in the analysis of characters and settings of a number of fiction stories that they will view, read or listen to. The product is a presentation of their ideas in a performance.

There are few opportunities to teach mathematics in this Rich Task. The analysis of characters could involve early pattern work (attribute discrimination) and the performance could involve location and direction. Table 5.4 summarises the mathematics in Task 4.

Table 5.4

Mathematics ideas for Year 1-3 Rich Task 4 Read and Talk About Stories

Prerequisite mathematics	Embedded mathematics	Associated mathematics
Patterns *(attribute recognition)*		Location/dir. – *location & direction*.

Task 5: Historical and Social Aspects of Craft

Students explore craft as a personal, social and cultural endeavour. The end product is an object made as an example of that craft which students will display at a stall prepared and run by them.

The mathematics involved in the project can be seen in the main steps of the Task. Exploring/selecting a craft involves discriminating between objects, shapes, colours and sizes. Designing an object involves 2D and 3D shapes and their role in artistic excellence. As well, discussion about the craft offers opportunities to look at length, area and mass and also symmetry; while the stall offers an opportunity for money (money handling and the concept of value) and money computation to be discussed. Table 5.5 summarises the mathematics in Task 5.

Table 5.5

Mathematics ideas for Year 1-3 Rich Task 5 Historical and Social Aspects of a Craft

Prerequisite mathematics	**Embedded mathematics**	**Associated mathematics**
Patterns *(attribute recognition)* Whole numbers	Shape - *3D shape & 2D shape*	Symmetry/transf. - *symmetry* Length Area Mass Add/subtr. - *concept, basic facts & computation* Money – *money handling & concept*

Summary

Tables 5.1 to 5.5 list the mathematics for each of the Years 1 to 3 Rich tasks. Combining these five tables identifies which of the mathematics ideas in the Years 1 to 3 part of the "summary syllabus" of Section 3 are prerequisite to, embedded in and associated with the five Rich Tasks. Table 5.6 summarises the relationship between mathematics content ideas and the Rich Tasks. The implications of this summary are in Section 6.

Table 5.6

Mathematics ideas present in Year 1-3 Rich Tasks

Mathematics idea	Prerequisite	Embedded	Associated
NUMBER Whole Numbers Fractions Decimals Percent/rate/ratio	**RT1; RT2; RT3; RT5**		
OPERATIONS Add/Subtr. Mult/Div.			**RT3** - *conc., basic f., comp.*
EARLY ALGEBRA Patterns Change/functions Equivalence	**RT4** - *attr recognition* **RT5** - *attr recognition*	**RT3**	**RT2**
SPACE Shape Location/Dir. Symmetry/Transf.		**RT5** **RT3**	**RT1; RT2; RT4**

MEASUREMENT Length Area Volume Mass Temperature Time Money		 **RT1** – *standard units* **RT1** – *standard units* **RT1** – *standard units* **RT1** - *standard units*	 **RT2** **RT2** **RT2**
CHANCE & DATA Chance Data	 **RT2** - *concept*	 **RT2** – *outcomes* **RT1; RT2; RT3**	

P – prerequisite mathematics; E – embedded mathematics; A – associated mathematics

5.2 Mathematics ideas for Year 4-6 Rich Tasks

For Years 4 to 6, the major mathematics ideas from the "summary syllabus" for the Tasks are as follows.

Task 1: Travel Itineraries

This Task requires students to design three alternative travel itineraries for the student and an exchange student, accompanied by an adult. The itineraries have to be justified in terms of costs and other reasons.

There is a great deal of mathematics that can be taught in this Task. Number (whole numbers to 999,999) is a prerequisite, while students need to know about: (a) computation (addition, subtraction, multiplication and division) with numbers and money, (b) time (diaries, timetables, time computations, etc.), (c) location/ direction (coordinates, maps), and (d) data (handling and display in tables, graphs etc.). There is also an opportunity to use the Task to teach: (a) decimal numbers (as the number framework for currencies), (b) percent (in terms of relationships between currencies, discounted fares, etc.), and (c) early algebra (change/functions, and equivalence – equals & equations) as part of the use of *spreadsheets*. Of course this also means that concepts, principles and basic facts for all operations need to be taught as prerequisites for computation and that common fractions need to be taught as prerequisites for decimals. Table 5.7 summarises the mathematics in Task 1.

Table 5.7

Mathematics ideas for Year 4 - 6 Rich Task 1 Travel Itineraries

Prerequisite mathematics	Embedded mathematics	Associated mathematics
Whole numbers Addn/subtn – *concepts, principles & basic facts* Multn/Divn – *concepts, principles & basic facts* Common factions	Addn/subtn - *computation* Multn/divn - *computation* Time Location/direction – *position* (coordinates) & *direction* (maps) Data – *handling & display*	Decimal numbers Percent/rate/ratio - *percent* Change/functions Equivalence – *equals & equations*

Task 2: Narrative: Away with Words

This Task requires students to perform some very specific tasks: categorise books, review one book and create an illustrated storybook for peers review.

The mathematics involved in this Task is basic: numbering of pages in a book (in creating a storybook), and sorting and classifying (in categorising books) – nothing at the level of Years 4 to 6. The only possibility is that shape and symmetry could be taught as a basis of illustrations. Table 5.8 summarises the mathematics in Task 2.

Table 5.8

Mathematics ideas for Year 4 - 6 Rich Task 2 Narrative Text

Prerequisite mathematics	Embedded mathematics	Associated mathematics
		Shape – *3D*, *2D*, *line* & *angle* Symmetry/transf - *symmetry*

Task 3: Personal Health Plan

Students are required to identify an aspect of their health or fitness that they want to improve, develop and implement a plan to improve it and evaluate the result.

There are some substantive and fundamental mathematics concepts to be taught in this Task. Students must know: (a) about whole numbers; (b) how to do all operations with these numbers, (c) about measurement (length, mass and time) to conduct their monitoring and evaluating of the plan, and (d) how to handle and display data in tabular and/or graphical forms. The teaching could be extended to cover number concepts that underlie measurement (e.g., decimals & percent– 100% fitness). This means that number ideas earlier in the teaching sequence (e.g. common fractions) need also to be covered. Table 5.9 summarises the mathematics in Task 3.

Table 5.9

Mathematics ideas for Year 4 - 6 Rich Task 3 Personal Health Plan

Prerequisite mathematics	Embedded mathematics	Associated mathematics
Whole numbers Common fractions	Length Mass Time Data - *handling & display*	Decimal numbers Percent

Task 4: A Celebratory, Festive or Artistic Event or Performance

In this Task students work in groups but in different capacities to plan, create and perform in an event or performance.

What constitutes essential mathematics varies with the nature of the event or performance selected by the students. For instance, a dance performance could involve patterns (of movement and sound) and could be used to motivate teaching of space, notably location/direction, more so than a singing recital. However, the task can be used to teach: (a) computation with money (organising the finances for staging an event or performances), and (b) location/direction (coordinates for ticketing). The measurement and space knowledge necessary for arranging the stage appear to be at a level below Years 4 to 6. Obviously, computation with money requires whole numbers and operations for concept, principles and basic facts as prerequisites and could be used to motivate decimal numbers (which then leads to common fractions being a prerequisite). The teaching of financial considerations could also be extended to the concept of percent (e.g., discounted tickets). There is also patterning involved in the staging of events and performances but not really at the level of Years 4 to 6. Table 5.10 summarises the mathematics in Task 4.

Table 5.10

Mathematics ideas for Year 4 - 6 Rich Task 4 A Celebratory, Festive or Artistic Event or Performance

Prerequisite mathematics	Embedded mathematics	Associated mathematics
Whole numbers Addn/subtn – *concepts*, *principles* & *basic facts* Multn/divn – *concepts*, *principles* & *basic facts* Money	Addn/subtn – *computation* Multn/divn – *computation* Location/direction – *position*	Common fractions Decimal numbers Percent

Task 5: Oral histories and Diverse and Changing Lifestyles

Students are required to interviews various members of their own community about changes in twentieth century lifestyles. They then use the recorded oral histories as the basis for an electronic media presentation.

The essential mathematics here is time (sequencing/selecting) and data (handling and display). The display component of data could be enhanced by, for example, inclusion of graphs, charts and tables in the electronic media presentation. Table 5.11 summarises the mathematics in Task 5.

Table 5.11

Mathematics ideas for Year 4 - 6 Rich Task 5 Oral Histories

Prerequisite mathematics	Embedded mathematics	Associated mathematics
	Time (*e.g., year, decade, century*) Data – *handling & display*	

Task 6: Design, Make and Display a Product

This task requires students to design, or improve the design, of a product. They will display the product publicly and create plan to mass produce and market the product.

There is some variation in the amount and kind of mathematics, depending on the product selected by the student. This notwithstanding, the Task requires students to use knowledge of: (a) computation with money (the marketing plan), (b) space – 2D and 3D shape (the basis of construction), and (c) measurement – length, volume, and mass (in construction and display). Chance could be part of the associated mathematics – it could be taught as part of the consideration of the most suitable materials for mass manufacture and also in developing the marketing plan (the best chance of succeeding). Prealgebra – equivalence could be part of associated mathematics if *spreadsheets* were used. There could also be a possibility of using the Task for some space idea, for example, location/direction in the preparation of the static display and symmetry/transformation in the construction of the working prototype.

This means that whole number and money is prerequisite for the computation along with concepts, principles and basic facts for the operations, and that common fractions and decimal numbers could be associated with the money and measurement. Table 5.12 summarises the mathematics in Task 6.

Table 5.12

Mathematics ideas for Year 4 - 6 Rich Task 6 Design, Make and Display a Product

Prerequisite mathematics	Embedded mathematics	Associated mathematics
Whole numbers Addn/subtn – *concepts, principles & basic facts* Multn/divn – *concepts, principles & basic facts* Money	Addn/subtn – *computation* Multn/divn – *computation* Shape – *3D & 2D* Length Volume – *solid* Mass	Equivalence – *equals & equations* Location/direction Symmetry/transformation Chance

Task 7: Space Futures

Students are introduced into the techniques and procedures of the mathematical and physical sciences. They are required to produce a model of the solar system, investigate the impact of space travel on earth and produce a design for an experiment to be performed on a spacecraft.

The essential mathematics here has to do with: (a) decimal numbers, (b) computation on these numbers, (c) location/direction; and (d) length, mass, temperature and time. This means that common fractions are a

prerequisite as are operation concepts, principles and basic facts. As well, shape (2D & 3D) could be motivated by the need to produce the dynamic model of the solar system.

Other mathematics could be involved in the Task depending on the experiment chosen. Overall, it is likely that chance could be motivated by the Task by looking at likelihood of different outcomes from these experiments. Table 5.13 summarises the mathematics in Task 7.

Table 5.13

Mathematics ideas for Year 4 - 6 Rich Task 7 Space Futures

Prerequisite mathematics	**Embedded mathematics**	**Associated mathematics**
Whole numbers Common fractions Addn/subtn – *concepts, principles & basic facts* Multn/divn – *concepts, principles & basic facts*	Decimal numbers Addn/subtn – *computation* Multn/divn – *computation* Location/direction Length Mass Temperature Time	Shape – *3D & 2D*

Summary

Tables 5.7 to 5.13 list the mathematics for each of the Years 4 to 6 Rich tasks. Combining these seven tables identifies which of the mathematics ideas in the Years 4 to 6 part of the "summary syllabus" of Section 3 are prerequisite to, embedded in and associated with the seven Rich Tasks. Table 5.14 summarises the relationship between mathematics content ideas and the Rich Tasks. The implications of this summary are in Section 6.

Table 5.14

Mathematics ideas present in Year 4 - 6 Rich Tasks

Mathematics idea	Prerequisite	Embedded	Associated
NUMBER			
Whole Numbers	**RT1; RT3; RT4; RT6; RT7**		
Fractions	**RT1; RT3; RT7**		**RT4**
Decimals		**RT7**	**RT1; RT3; RT4**
Percent/rate/ratio			**RT1** –*percent*; **RT3** –*percent*; **RT4** - *percent*
OPERATIONS			
Add/Subtr.	**RT1** – *concept, pr, basic f*; **RT4** – *concept, p, basic f*; **RT6** – *concept, pr, basic f*; **RT7** – *concept, pr, basic f*	**RT1** – *computation*; **RT4** – *computation;* **RT6** – *computation*; **RT7** – *computation*	
Mult/Div.	**RT1** – concepts, pr, *basic f;* **RT4** – concepts, pr, *basic f;* **RT6** – *concept, pr, basic f*; **RT7** – *concept, pr, basic f*	**RT1** – *computation*; **RT4** – *computation*; **RT6** – *computation*; **RT7** – *computation*	
EARLY ALGEBRA			
Patterns			
Change/functions			**RT1**

Equivalence			**RT1; RT7**

SPACE Shape Location/Dir. Symmetry/Transf.		 **RT6** **RT1;** **RT4** – *position* 	 **RT2; RT7** **RT7** **RT2** – *symmetry*; **RT7**
MEASUREMENT Length Area Volume Mass Temperature Time Money	 **RT4; RT6**	 **RT3; RT6; RT7** **RT6** - *solid* **RT3; RT6; RT7** **RT7** **RT1; RT3; RT5; RT7** 	
CHANCE & DATA Chance Data		 **RT1; RT3; RT5**	 **RT7**

P – prerequisite mathematics; E – embedded mathematics; A – associated mathematics

5.3 Mathematics ideas for Year 7-9 Tasks

For Years 7 to 9, the major mathematics ideas from the "summary syllabus" for the Tasks are as follows.

Task 1: Science and Ethics Confer

Select an area of biotechnology, identify techniques and significant recent contributions, identify an ethical issue or question associated with this area, and prepare pre-conference materials for an international conference that features 6 selected speakers who can showcase advancements which might resolve the ethical objections.

In general, there appears to be no embedded mathematics in this Task and mathematics will only be associated with the Task if it is part of the biotechnology area chosen. There is of course, significant communication and argumentation in the Task. Table 5.15 summarises the mathematics in Task 1.

Table 5.15

Mathematics ideas for Year 7- 9 Rich Task 1 Science and Ethics Confer

Prerequisite mathematics	Embedded mathematics	Associated mathematics
		Depends on Biotechnology area chosen.

Task 2: Improving Wellbeing in the Community

This Task requires students to work with a local community to improve an aspect of its wellbeing. The students have to develop, implement, enact and evaluate the plan and make recommendations for future actions.

Once again, the mathematics in this Task depends on the local community project chosen. It is possible for the project to involve significant measurement or space activity. However, in general, the mathematics embedded in the Task involves data handling and display (e.g., designing surveys, collecting and analysing data, drawing inference). There may also be an opportunity to use the Task to introduce Chance – outcomes because students will have to make predictions based on the likelihood of success of alternative courses of action. This obviously requires Chance – concepts as a prerequisite. Table 5.16 summarises the mathematics in Task 2.

Table 5.16

Mathematics ideas for Year 7 - 9 Rich Task 2 Community Wellbeing

Prerequisite mathematics	Embedded mathematics	Associated mathematics
Chance – *concept*	Data – *handling & display*	Chance - *outcomes* Other mathematics depends on project chosen

Task 3: The Built Environment: Designing a Structure

Students identify a client's needs and design and present a structure, taking into account environmental, aesthetic and cost considerations.

The Task gives teachers the opportunity to teach a range of mathematics concepts as embedded mathematics: (a); computation in all four operations and financial management in money are required to determine costs (b) space activities in shape (3D and 3D shape, line and angle) and location/direction – position (particularly scale drawings) are required to design and present structure; and (c) measurement of length, area, volume and mass are required to design the structure. To undertake all this mathematics requires some prerequisite knowledge: (a) knowledge in decimal numbers underlies measurement and money; and (b) knowledge of ratio underlies scale drawings. There are also some possibilities that algebra and data could be motivated by the Task, but the connection is tenuous and the work involved is below Years 7 to 9, so no associated mathematics is included. Table 5.17 summarises the mathematics in Task 3.

Table 5.17

Mathematics ideas for Year 7- 9 Rich Task 3 Built Environment

Prerequisite mathematics	Embedded mathematics	Associated mathematics
Decimal numbers Percent/rate/ratio – *rate & ratio*	Addition/subtraction - *computation* Multiplication/division - *computation* Shape – *3D*, *2D*, *line & angle* Location/direction - *position* Length Area Volume - *solid* Mass Money	

Task 4: Australian National Identity: Influences and Perspectives

Students will create, produce and present a filmed documentary/docudrama showing their understanding of the different influences and perspectives on the Australian national identity as found from research into the past history of Australia and interviews with local people who have been involved in or studied this history.

The embedded mathematics here mostly has to do with data handling such as interviewing, surveying, recording, noting, sorting, and classifying. It also has aspects of the data display with respect to interpreting and making inferences from the gathered data. There is scope to extend ore into data display if the presentation of the data in the filmed documentary includes the use of mathematical forms of display such as pie charts, line graphs and histograms. Time also forms part of the Task in dealing with different periods of time; along with location/direction which provides a framework for the movement needed in setting up the documentary or docudrama. However, the components of time and location/direction used are from Years 4 to 6 not Years 7 to 9. Table 5.18 summarises the mathematics in Task 4.

Table 5.18

Mathematics ideas for Year 7-9 Rich Task 4 Australian National Identity

Prerequisite mathematics	Embedded mathematics	Associated mathematics
	Data - *handling*	Data - *display*

Task 5: Personal Career Development Plan

Students are required to undertake a career planning process. They will develop their own individual career plan, including a resume, taking into account the various work options and their own interests and strengths.

The mathematics directly involved in the Task is data handling and display (including flowcharts and timelines). Time is a prerequisite for this but at Years 4 to 6 level not 7 to 9. However, as described in Section 4.3, there is areal opportunity to use the task to teach chance, particularly with respect to evaluating the expectations of various work options and predicting future events. Thus chance – outcomes is associated with the Task; this means that chance – concept is a prerequisite. Table 5.19 summarises the mathematics in Task 5.

Table 5.19

Mathematics ideas for Year 7 - 9 Rich Task 5 Personal Career Development Plan

Prerequisite mathematics	Embedded mathematics	Associated mathematics
Chance – *concept*	Data – *handling & display*	Chance - *outcomes*

Task 6: Opinion-making Oracy

This task involves students making forceful speeches on an issue of national or international significance to three different audiences in different forums.

The mathematics in this Task would seem to depend on the issue chosen. However, the preparation of the speeches by the students does offer some opportunities for mathematics; the speeches could be made more powerful with collection, organisation and interpretation of data during the research for the speeches and with the inclusion of graphical and tabular data in the speech. Thus, data could be associated with the Task. Table 5.20 summarises the mathematics in Task 6.

Table 5.20

Mathematics ideas for Year 7 - 9 Rich Task 6 Opinion-making Oracy

Prerequisite mathematics	Embedded mathematics	Associated mathematics
		Data – *handling & display*

Task 7 *Pi in the Sky**

Students are required to demonstrate an understanding of the mathematical approaches used to frame and answer questions about astronomy in cultures from three different historical ages. They have to communicate their ideas in a lesson to a selected audience.

This Task calls for a good understanding of the various branches of mathematics: (a) number – rate and ratio (which leads to whole numbers, common fractions and multiplication/divison being prerequisites);(b) space – coordinate systems, scale drawings (which leads to ratio and proportion being prerequisites) and plotting position and direction; and (c) measurement – length, area, solid volume, mass, temperature & time (which because of the largeness of the distances leads to Years 7 to 9 whole and decimal numbers in exponential notation being prerequisites). Obviously, computation in addition/subtraction is also a prerequisite but at Years 4 to 6 level not 7 to 9 level. As well, trigonometry could be motivated by the Task. Table 5.21 summarises the mathematics in Task 7.

Table 5.21

Mathematics ideas for Year 7 - 9 Rich Task 7 Pi in the Sky

Prerequisite mathematics	Embedded mathematics	Associated mathematics
Whole numbers Common fractions Decimal numbers Multiplication/division - *computation*	Percent, rate and ratio Location/dirn – *position & direction* Length Area Volume - *solid* Mass Time	Angle

Note: The alternative Task to RT7, *The Shape We're In*, has not been analysed as it was still in a draft form when this analysis was conducted.

Task 8: International Trade

This task requires students to demonstrate their knowledge of international trade as well as a language other than English. The knowledge of international trade is employed conduct a detailed analysis of an export opportunity; the skills in a language other than English are used to promote the export opportunity to potential buyers and backers in that language.

The mathematics embedded in this Task is percent (e.g., interest, excise, duty, tax), rate and ratio (e.g., conversion rates), and money (i.e., financial mathematics). Obviously common fractions and decimal numbers are prerequisites to percent, rate and ratio and to financial mathematics respectively. There is also an opportunity for chance to be associated with this Task as it is the basis of determining whether an "export opportunity" is worth supporting. As well, time zones and their effect on the opportunity could be included. Table 5.22 summarises the mathematics in Task 8.

Table 5.22

Mathematics ideas for Year 7- 9 Rich Task 8 International Trade

Prerequisite mathematics	Embedded mathematics	Associated mathematics
Common fractions Decimal numbers	Percent/rate/ratio Money (*financial mathematics)*	Time - *concept* Chance – *concept & outcomes*

Summary

Tables 5.15 to 5.22 list the mathematics for each of the Years 7 to 9 Rich tasks. Combining these eight tables identifies which of the mathematics ideas in the Years 7 to 9 part of the "summary syllabus" of Section 3 are prerequisite to, embedded in and associated with the eight Rich Tasks. Table 5.23 summarises the relationship between mathematics content ideas and the Rich Tasks. The implications of this summary are in Section 6.

Table 5.23

Mathematics ideas present in Year 7 - 9 Rich Tasks

Mathematics idea	Prerequisite	Embedded	Associated
NUMBER			
Whole Numbers	**RT7**		
Fractions	**RT7; RT8**		
Decimals	**RT3; RT7; RT8**		

Percent/rate/ratio	**RT3** – *rate & ratio*	**RT7; RT8**	

Section 6:
Teaching mathematics with Rich Tasks

The main question for this report was how Rich Tasks can be an effective vehicle for teaching, learning and assessing mathematics. Four tasks were set for the report as follows.

(5) Development of a mathematics "summary syllabus" representing best international practice in terms of mathematics teaching and learning in Years 1 to 9.

(6) Theoretical analysis of knowledge from this "summary syllabus" required for and engendered by the Rich Tasks.

(7) Development of plans for integrating the Rich Tasks with the "summary syllabus" for the benefit of mathematics teaching, learning and assessment.

(8) Publication of a report outlining findings from the above tasks.

Tasks (1) and (2) have been completed in Sections 3 and 5. This Section will focus on developing plans for Years 1 to 3, 4 to 6 and 7 to 9 that integrate the Rich Tasks with mathematics.

6.1 Implications from analysis of Rich Tasks

The discussion and analysis in Sections 4 and 5 showed that not all mathematics process and content ideas can be developed within the Rich Tasks.

For the mathematics process ideas, the following was evident.

(6) The *argumentation* and *mathematisation* process ideas are supported by the Rich Tasks.

(7) The *connections* and *strategic* process ideas have to be developed outside the Rich Tasks but the Tasks do offer some opportunities for considering and applying these ideas.

(8) The *abstraction* process idea appears to be difficult for the Rich Tasks. In fact, the Tasks appear to contain few *strategic* connections and strategic process ideas. Abstraction is a particular difficulty for Rich Tasks process ideas.

For the mathematics content ideas, as Tables 5.6, 5.14 and 5.23 showed, many important mathematics topics were not required for any Rich Tasks or were only associated with Rich Tasks. This is summarised as follows.

(1) Years 1 to 3. Early activity in the number ideas of *fractions* and *decimals*, the operations of *multiplication/division*, the early algebra topics of *change/functions* and *equivalence*, the space ideas of *symmetry/transformations*, the measurement topics of *area*, *volume* and *money*, are in no Rich Task. The operations of *addition/subtraction* are only associated with one Task. *Whole numbers* is only a prerequisite but in 4 of the 5 Tasks. *Patterns*, *shape*, *location/direction*, *length*, *mass*, *temperature*, *time*, *chance* and *data* are embedded in Tasks.

(2) Years 4 to 6. *Patterns*, *area* and liquid *volume* are not in any Rich Task. Early *percent*, early algebra idea of *change/functions* and *equivalence*, *symmetry/transformations* and *chance* are only in Associated mathematics. *Whole numbers*, *fractions*, *operations* at concept, principles and basic fact levels and *money* are only in Prerequisite mathematics (or Prerequisite and Associated, in fractions case). Only *decimals*, computation in *operations*, *shape*, *location/direction*, *length*, solid *volume*, *mass*, *temperature*, *time*, *chance* and *data* are embedded in a Rich Task.

(3) Years 7 to 9. Early algebra (i.e., *patterns*, *change/functions* and *equivalence*), the space topic of *symmetry/transformations* and liquid *volume* are not in any Rich Task. *Chance* is only in Prerequisite and Associated mathematics. *Whole numbers*, *fractions* and *decimals* are only Prerequisites. *Percent/rate/ratio*, *computation* in operations, *shape*, *location/direction* and measurement topics of *length*, *area*, solid *volume*, *mass*, *temperature*, *time* and *money*, *chance* and *data* are embedded in a Task.

The pattern of absence and presence of mathematics content in Rich Tasks repeats what was discussed in Issue 1 in Section 4; that is, that:

- Rich Tasks *support mathematics that is related to investigation and presentation*; and
- Rich Tasks *do not appear to support abstraction* (i.e., concept development).

The result of this is the predominance of whole numbers, early patterns, shape and location/direction, measures such as length, mass, temperature and time, and chance and data in Rich Tasks. It also means that much of the crucial concept development for place value, fractions, operations, algebra, some aspects of space, and the early development of measurements (attribute, comparison and non-standard units) are not in any Rich Task.

6.2 Plans for integrating Rich Tasks with mathematics

Rich Tasks act as a vehicle for assessment and reporting as well as teaching. As such, they are to be undertaken predominantly in the last year of each three-year period, Years 3, 6 and 9 (the juncture years). One way in which this could be achieved is to consider each three-year period as a sequence from *introduction* (the first year) to *abstraction* (the second year) to *application* (the third year). This would mean the following.

(1) Introduction. The first year of each three-year period (Years 1, 4 and 7) would focus on using a variety of representations to introduce the major mathematics ideas in the three-year period as rich concepts.

(2) Abstraction. The second year of each three-year period (Years 2, 5 and 8) would focus on developing structural understanding for the mathematics ideas at the symbolic level.

(3) Application. This year would focus on applying the ideas developed in the previous two years to, including to the Rich Tasks.

This could not be a rigid sequence – there would have to be some introduction, abstraction and application in each year. And it is only necessary for introduction to be early in the three-year cycle for most mathematics while application is late. However, because of the relationship of the mathematics to the three-year periods, it appears possible for much of the mathematics as the following examples show.

(1) Year 1 obviously begins the mathematics sequence – the notion of number and place value and the operations can be begun at least in the first two years. Similarly, Years 1 and 2 begin the mathematics in early algebra, space, much of measurement, and chance and data.

(2) Year 4 marks the beginning of fractions and decimals, although some is in Year 3, more formal development of pattern and change rules and backtracking and balance in early algebra, classifications and properties in space, metrics in measurement (e.g., standard units) with the move to decimals, and many sample spaces and complex graphs and charts in chance and data.

(3) Year 7 begins to move to systems of number and to percent, rate and ratio in number, rational numbers in operations, formal use of letters in early algebra, systems and relationships in space, formulae in measurement, and theories, experiments and investigations in chance and data.

Table 6.1 provides a summary of the placement of Rich Tasks in relation to mathematics content ideas from the "summary syllabus". Content ideas that are related to Rich Tasks are denoted by P, E and A for prerequisite, embedded and associated mathematics respectively.

Table 6.1

Placement of Rich Tasks for Years 1 to 3, 4 to 6 and 7 to 9

Year	Term 1	Term 2	Term 3	Term 4
YEARS 1 to 3				
1	Early number Early space – 3D & 2D shape Measurement - time	Number continued Early algebra (Patterns/Change) Space – locn/dirn Measurement – length & temperature	Early operation (+/-) Early chance & data Measurement – area & volume	Number – early grouping Operations +/-; early x/÷ Early algebra – equals Space - symmetry
2	Number – grouping & early place value Early algebra – patterns & change Space – 2D and 3D shape Measurement – temperature & time	Number – 2 digits Operations +/-; early x/÷ Space – line & angle Measurement – length & mass Chance	Operations +/- to 2 digit computation Early algebra – equals & equations Measurement - area & volume Data - handling	Number & operations continued Space - locn/dirn (A-RT4) & symm/transf Data -display **RT4**
3	Number – 3 digits (P-RT5/2) Operations – concepts, pr & basic facts Early algebra pattern & change rules Shape (E-RT5) **RT5**	Number – early fractions Operations – to 3 digit computation Chance (P/E-RT2) Data (E-RT2) Measurement (A-RT2) **RT2**	Number – 3 digits (P-RT3) Location/dirn (A-RT2, E-RT3) Data (E-RT3) **RT3**	Number – early fractions/decimals Operations - x/÷ Location/dirn (A-RT1) Measurement (E-RT1) Data (E-RT1) **RT1**
YEARS 4 to 6				
4	Number – whole nos & fractions Operations – concepts, pr, basic facts & computation	Early algebra – patterns, change & equivalence Space – shape, locn/dirn & symm/transf Measurement – length, mass, temperature & time	Number –fractions & decimals Operations - computation Chance & data	Number – decimals Space – shape, line & angle Measurement – area & volume
5	Number, whole numbers & fractions Operations – basic facts Space – shape & locn/dirn	Number - decimals Operations – computation (E-RT4) Early algebra – patterns/change Measurement - length, mass, temperature & time	Early algebra – equiv Space – shape & symm/transf (A-RT2) Measurement – area & volume Chance **RT2**	Number – whole nos, fract & decimals Measurement – time (E-RT5; A-RT1) Data (E-RT5/1) **RT5**
6	Number – whole nos, fract, decimals & percent (A-RT1/3) Operations – basic f & computation (E-RT1/6) Space – locn/dirn (E-RT1/7) **RT1**	Early algebra – equivalence (A-RT6) Space – shape (E-RT6) & symm/transf (A-RT6) Measurement – length, volume, temperature, time (E-RT6/3/7) **RT6**	Number – whole nos & fractions (P-RT3/7), and decimals & percent (A-RT3; E-RT7) Operations – computation Data (E-RT3) **RT3**	Number – fract, decimals (A-RT4) & percent (A-RT4) Operations – computation (E-RT7/4) Space – locn/dirn (E-RT4) **RT4/RT7**

YEARS 7 to 9				
7	Number – whole nos, fract, dec & percent Operations – basic f & computation Early algebra – patterns, change & functions	Operations – computation Space – shape & locn/dirn Measurement – length, area & volume Chance	Number – fract, dec & percent Measurement – mass, temperature, time & money Data	Operations – computation Early algebra – equivalence Space – shape, line, angle & symm/transf
8	Number – whole nos, fract, decimals & percent Operations – basic f & computation Space – shape & angle (P-RT7) Measurement – length, area, volume, mass, time	Number – rational nos & negatives Early algebra – patterns, change & fns, & equivalence Space – locn/dirn & symm/transf	Number – fract & dec (P-RT8) & percent/rate /ratio (P-RT8) Chance & data (P/E-RT2) Measurement – area & volume **RT2**	Operations – computation Measurement – money (E-RT8) & Time (A-RT8) Chance (A-RT8) **RT8**
9	Number – fract, decimal (P-RT3), scientific notation & negatives Early algebra - patterns, change & fns, & equivalence **RT6**	Operation – computation (E-RT3) Space – shapes, lines & angles (trig) & locn/dirn (E-RT3) Measurement - length, area, volume, mass, temperature, time (E-RT3) **RT3**	Number – fract, dec & percent (P-RT7) Operations – computation (P-RT7) Measurement – money Chance (P/A-RT5) Space – symm/transf Data (E/A-RT4/5) **RT1/RT4**	Number – percent/rate /ratio (E-RT7) Space – angle (A-RT7) & locn/dirn (E-RT7) Measurement – length, area, volume, mass & time (E-RT7) **RT5/RT7**

6.3 Conclusions and implications from the report

The major implications from the report are as follows.

(6) The Rich Tasks are inadequate on their own to teach, assess or report on mathematics learning. Analysis shows that major process and content ideas are not covered by the Tasks.

(7) Although considering mathematics as prerequisite for, embedded in and associated with Rich Tasks increases the mathematics that can be taught in relation to the Tasks, there are major components of mathematics that must be taught outside of the Rich Tasks.

(8) This out-of-Task mathematics needs to focus predominantly on concept development that leads to abstraction or mathematics topics that are not normally central to investigation and presentation.

(9) The Rich Tasks can be integrated with mathematics if out-of-Task, prerequisite, embedded and associated mathematics is sequenced to enable both Tasks and mathematics to be developed.

(10) This integration also requires that Rich Tasks be built into the curriculum and part of a general approach to teaching and learning that focuses on authentic situations and intellectual engagement.

Section 7:
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